Tutorial exercises, Feburary 14

1. (This is Question 5.3/5.4 in the text book)

   A plumb line is held steady while being carried along in a moving train. The train is accelerating with a constant acceleration $g/10$. The mass of the plumb bob is m and you may ignore any effects of the Earth's rotation.

   (a) Find the tension in the cord and the deflection angle from the local vertical.

   (b) if the plumb bob is made to oscillate like a simple pendulum of length l, find the period of oscillation for small amplitude.

Answers:

(a) In the accelerating frame, the force plumb bob $m\boldsymbol{a}' = m\boldsymbol{g} - m\boldsymbol{A}_0$ is opposed by the tension $\boldsymbol{S}$. Therefore

$$\boldsymbol{S} = -m\boldsymbol{g} + m\boldsymbol{A}_0.$$

and since the acceleration is horizontal, $\boldsymbol{g} \cdot \boldsymbol{A}_0 = 0$. The magnitude of the tension is

$$S = \sqrt{\boldsymbol{S} \cdot \boldsymbol{S}} = m\sqrt{(-\boldsymbol{g} + m\boldsymbol{A}_0) \cdot (-\boldsymbol{g} + m\boldsymbol{A}_0)} = m\sqrt{g^2 + A_0^2} = mg\sqrt{1 + (1/10)^2} = mg\sqrt{1.01} \simeq 1.0499$$

The deflection angle ϵ is given by

$$\boldsymbol{g} \cdot \boldsymbol{S} = -gS\cos\epsilon = mg^2\sqrt{1.01}\cos\epsilon$$

so

$$\cos\epsilon = \frac{\boldsymbol{g} \cdot \boldsymbol{S}}{gS} = \frac{mg^2 + 0}{mg^2\sqrt{1.01}} \simeq 0.9950$$

and therefore $\epsilon \simeq 5.7^\circ$.

(b) The apparent gravitation acceleration is

$$g' = S/m = g\sqrt{1.01}$$

so the period is

$$T = \sqrt{l/g'} \simeq 0.9950\sqrt{l/g}.$$