1 Tutorial exercises, January 17

1. For an object moving in one dimension,
   (a) suppose that the force acting on a article is the product of a function of position and a function of velocity: $F(x, v) = f(x)g(v)$. Show that the equation of motion can be solved by integration.
   (b) If instead the force is a product of a function of position and a function of time, can the equation of motion be solved by simple integration?
   (c) If the force is a product of a function of vecocity and a function of time, can the equation of motion be solved by simple integration?

Answers:

1. (a) The equation of motion is

$$m\ddot{x} = f(x)g(v) = m\frac{d}{dt}v = mv\frac{dv}{dx}. \tag{1.1}$$

We can separate the terms involving x and v as follows,

$$f(x)dx = m\frac{vdv}{g(v)}. \tag{1.2}$$

We can now integrate both sides.

$$\int f(x)dx = m\int \frac{vdv}{g(v)}, \tag{1.3}$$

This gives a relation between v and x. If we can solve the resulting equation for the velocity $v(x)$, we can integrate it to get the time as a function of x,

$$t = \int \frac{dx}{v(x)}. \tag{1.4}$$

(b) In this case,

$$m\ddot{x} = f(x)g(t) = m\frac{d}{dt}\frac{dx}{dt} \tag{1.5}$$

and there is no general way to separate the variables.

(c) Now we have

$$m\ddot{x} = f(v)g(t) = m\frac{dv}{dt}. \tag{1.6}$$

Here we can separate the variables,

$$m\int \frac{dv}{f(v)} = \int g(t)dt. \tag{1.7}$$

The integration gives a relationship $v(t)$ between velocity and time. Assuming that we can solve this equation for $v(t)$, We can then find x by integrating the velocity,

$$x = \int v(t)dt. \tag{1.8}$$