Tutorial exercises, March 21

1. A horizontal force F is applied to the axle of a wheel of radius a, which starting from rest at $x = 0$ accelerates along a flat horizontal surface.

   (a) Assuming that the wheel rolls without slipping, find the kinetic energy of the wheel as a function of the horizontal velocity $\dot{x}$.

   (b) Equate this energy to the work done by the force F, and by differentiating the result, find an equation for the acceleration $\ddot{x}$.

Answers:

1. (a) The energy of motion the centre of mass (CM) is $m\dot{x}^2/2$ and the energy of the rotational motion is $I\omega^2/2$. Therefore the total energy is

$$E = \frac{1}{2}m\dot{x}^2 + \frac{1}{2}I\dot{\omega}^2$$

Since the wheel does not slip, the linear and angular velocities are related by $\dot{x} = a\omega$. Therefore,

$$E = \frac{1}{2}(m + I/a^2)\dot{x}^2 = \frac{1}{2}m(1 + k^2/a^2)\dot{x}^2$$

where $k^2 = I/m$.

(b) Equating this to the work done,

$$W = \int F dx = Fx = \frac{1}{2}m(1 + k^2/a^2)\dot{x}^2.$$

Now take the derivative with respect to time,

$$F\dot{x} = m(1 + k^2/a^2)\dot{x}\ddot{x}.$$

Solving this for the acceleration,

$$\ddot{x} = \frac{F}{m(1 + k^2/a^2)} \tag{1}$$

We see that the wheel behaves as if it has a mass that is greater by a factor $(1 + k^2/a^2)$.