Simplified Solution to Homework set II Q.1.

1.1) for $l=0$, we have

$$\tan\delta_0(k) = \frac{j_0'(\rho)}{n_0'(\rho)} \frac{\left[1 - \tilde{\rho}\frac{j_0'(\tilde{\rho})}{j_0(\tilde{\rho})} \Big/ \rho\frac{j_0'(\rho)}{j_0(\rho)}\right]}{\underbrace{\left[1 - \tilde{\rho}\frac{j_0'(\tilde{\rho})}{j_0(\tilde{\rho})} \Big/ \rho\frac{n_0'(\rho)}{n_0(\rho)}\right]}} \Bigg|_{\substack{\rho = k r_0 \\ \tilde{\rho}^2 = \rho^2 + 2B}}$$

we expand the RHE near $\rho \simeq 0$

$$\frac{j_0'(\rho)}{n_0'(\rho)} \simeq -\frac{1}{3}\rho^3, \quad \frac{\rho j_0'(\rho)}{j_0(\rho)} = -\frac{1}{3}\rho^2$$

$$\underbrace{1 - \tilde{\rho}\frac{j_0'(\tilde{\rho})}{j_0(\tilde{\rho})} \Big/ \rho\frac{n_0'(\rho)}{n_0(\rho)}}_{I(\rho)} = I(\rho=0) + \frac{1}{2}\frac{\partial^2 I(\rho)}{\partial\rho^2}\Big|_{\rho=0} \rho^2$$

(linear term vanishes!!)

then $$\tan\delta_0(k) \simeq \frac{-\rho_0 J(\rho_0)}{I(0) + \frac{1}{2}I''(\rho)\big|_{\rho=0}\rho_0^2} \quad (\rho_0 = k r_0)$$

$J(\rho_0) = J(0) + \frac{J''}{2}\rho_0^2 + \cdots$; Near Resonance, $I(0)^{-1} \to \infty$.

so: $$\tan\delta_0(k) \simeq -\frac{k r_0 \frac{J(0)}{I(0)}}{1 + \frac{1}{2}\frac{I''(0)}{I(0)}\rho^2}, \quad a_s = r_0\frac{J(0)}{I(0)}$$

$$= -\frac{k a_s}{1 + k^2 r_0 a_s \times \frac{1}{2}\frac{I''(0)}{J(0)}}$$

$I''(0)$, $J(0)$ computable.

(See Solution II for more systematic discussions) bound state solution

<u>Erratum</u> to $\bar{h}_\ell(\rho)$ on page 81

$$\bar{h}_\ell(\rho) = \frac{(2\ell-1)!!}{\rho^{\ell+1}} + \left[-\frac{1}{2}\right](2\ell-3)!!\,\frac{1}{\rho^{\ell-1}} + \cdots$$

$$= \frac{(2\ell-1)!!}{\rho^{\ell+1}}\left[1 - \frac{1}{2(2\ell-1)}\rho^2 + \cdots\right]$$

<u>Correction $O(\rho^2)$ NOT $O(\rho^{2\ell})$.</u>