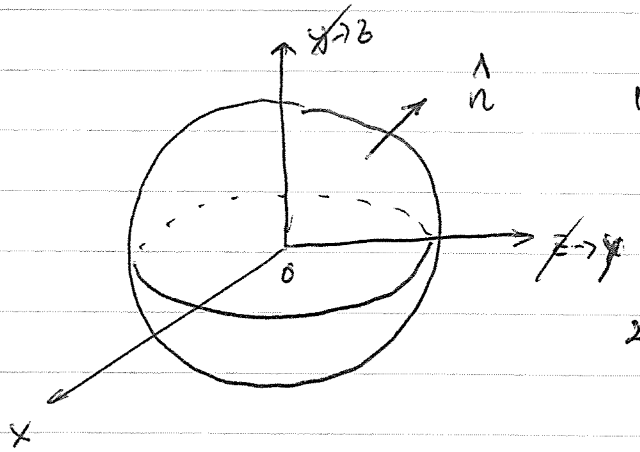
Supplementary Materials

Stuff on Spin projection and Spin Rotation you shall know

Bloch Sphere for Spinors : unit Vector $\hat{n} = \hat{n}(\theta, \varphi)$

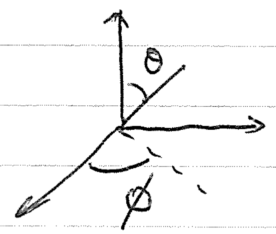


1) Spinor pointing along $\hat{n}$ if opposite

$$\vec{\sigma}\cdot\hat{n}\,|\chi(\hat{n})\rangle_{\pm} = (\pm 1)\,|\chi(\hat{n})\rangle_{\pm}$$

2) Spinor Reflection symmetry

$$\chi_{+}(\hat{n}) = \chi_{-}(-\hat{n})$$

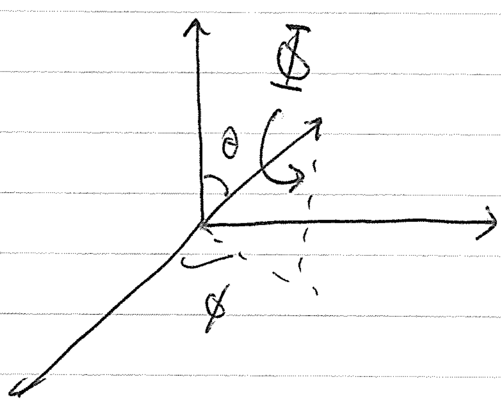


$$\chi_{+}(\hat{n}) = \begin{bmatrix} \cos\frac{\theta}{2} \\ \sin\frac{\theta}{2}\, e^{i\varphi} \end{bmatrix} \quad \text{(up to a gauge)}$$

Spin Rotations :

Spinor Rotated along $\hat{n}$ axis with angle Φ by

$$U(\hat{n}, \Phi) = e^{-i\frac{\vec{\sigma}\cdot\hat{n}}{2}\Phi} = \cos\frac{\Phi}{2} - i\,\vec{\sigma}\cdot\hat{n}\,\sin\frac{\Phi}{2}$$

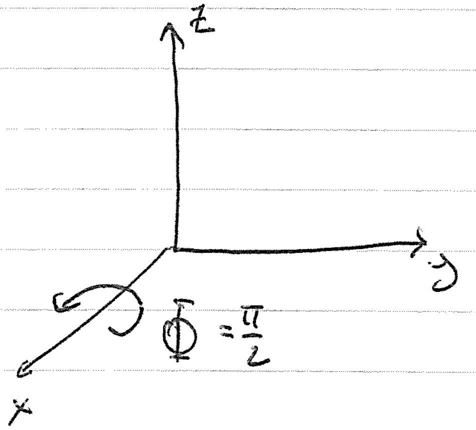


* Note $\hat{n}$ now is the Rotation axis as in the Euler Rotation SO(3).

* Φ is the angle set by the right hand rule.

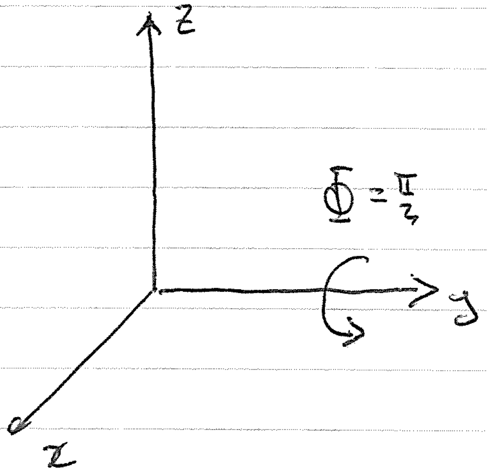
Example

Pauli Matrices or SU(2) Rotations form a quaternion Representation for SO(3). (Hamilton)



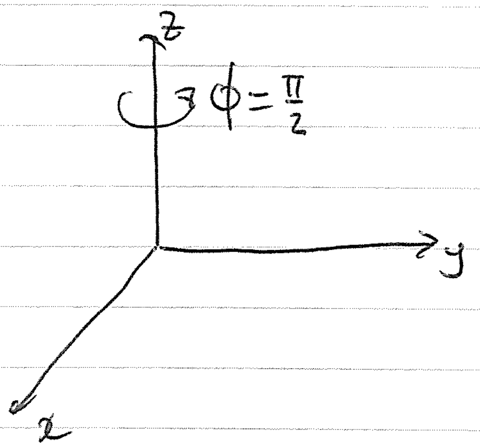
$$U(\hat{e}_x, \tfrac{\pi}{2}) = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 & -i \\ -i & 1 \end{bmatrix}$$

$$\begin{cases} U(\hat{e}_x, \tfrac{\pi}{2})\, \chi_+(\hat{e}_z) = \chi_-(\hat{e}_y) \\ U(\hat{e}_x, \tfrac{\pi}{2})\, \chi_+(\hat{e}_y) = \chi_+(\hat{e}_x) \end{cases}$$



$$U(\hat{e}_y, \tfrac{\pi}{2}) = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix}$$

$$\begin{cases} U(\hat{e}_y, \tfrac{\pi}{2})\, \chi_+(\hat{e}_z) = \chi_+(\hat{e}_x) \\ U(\hat{e}_y, \tfrac{\pi}{2})\, \chi_+(\hat{e}_x) = \chi_-(\hat{e}_z) \end{cases}$$



$$U(\hat{e}_z, \tfrac{\pi}{2}) = \frac{1}{\sqrt{2}}\begin{bmatrix} 1-i & 0 \\ 0 & 1+i \end{bmatrix}$$

$$\begin{cases} U(\hat{e}_z, \tfrac{\pi}{2})\, \chi_+(\hat{e}_x) = \chi_+(\hat{e}_y)\,[e^{-i\frac{\pi}{4}}] \\ U(\hat{e}_z, \tfrac{\pi}{2})\, \chi_+(\hat{e}_y) = \chi_-(\hat{e}_x)\,[e^{-i\frac{\pi}{4}}] \end{cases}$$

associate with Berry's phase

Note SU(2) Rotation $\boxed{U(\hat{n}, \Phi = 2\pi) = -1}$

Characteristic of Spin-1/2 particles

$U(\hat{n}, \Phi = 4\pi) = \mathbb{1}$ → <u>double-Coverage of SO(3)</u>