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Supplementary Mat. on "T" matrix for pseudo potential (S-wave)

$$V_{\vec{k}_f,\vec{k}_i} = \begin{cases} \underbrace{g}_{\text{strength}}, & |\vec{k}_f|, |\vec{k}_i| \le \Lambda \\ 0, & |\vec{k}_f| \text{ or } |\vec{k}_i| > \Lambda \end{cases}$$

$$T_{\vec{k}_f,\vec{k}_i}(E) = V_{\vec{k}_f,\vec{k}_i} + \frac{1}{\Omega}\sum_{\vec{k}'} V_{\vec{k}_f,\vec{k}'} G_E(\vec{k}') T_{\vec{k}',\vec{k}_i}(E)$$

$$= V_{\vec{k}_f,\vec{k}_i} + \int \frac{d^d\vec{k}'}{(2\pi)^d} V_{\vec{k}_f,\vec{k}'} \frac{1}{E - \frac{k'^2}{2} + i\delta} T_{\vec{k}',\vec{k}_i}(E)$$

$V_{\vec{k}_f,\vec{k}_i} = \Omega \langle \vec{k}_f | V | \vec{k}_i \rangle$ ← Matrix interaction

For pseudo potential, $T_{\vec{k}_f,\vec{k}_i}(E) = T(E)$

$$T(E) = g + g\left[\int \frac{d^3\vec{k}'}{(2\pi)^3} \frac{1}{E - \frac{k'^2}{2} + i\delta}\right] T(E)$$

→ Complex function of "E"

or
$$T(E) = \frac{1}{\frac{1}{g} + \int^{\Lambda} \frac{d^3\vec{k}'}{(2\pi)^3} \frac{2}{k'^2 - k_E^2 - i\delta}}$$

$$\frac{k_E^2}{2} = E$$

$$= \frac{1}{\underbrace{\frac{1}{g} + \frac{1}{\pi^2}\Lambda}_{} + i\frac{k_E}{2\pi}} = \frac{1}{\frac{1}{2\pi a_{sc}} + i\frac{k_E}{2\pi}}$$

so that

$$f(\theta) = -\frac{2}{4\pi} T(E) = -\frac{a_{sc}}{1 + ik_E a_{sc}} = \frac{\sin\delta_0}{k} e^{i\delta_0}$$

Note that
We identify $\frac{1}{2\pi a_{sc}} = \frac{1}{g} + \frac{1}{\pi^2}\Lambda$ so that $\boxed{\lim_{k\to 0} \frac{\delta_0}{k} = -a_{sc}}$

$$T(E) = \frac{2\pi a_{sc}}{1 + i k_E a_{sc}}$$ ← One of the well-known result.

<u>Technical Remark:</u>

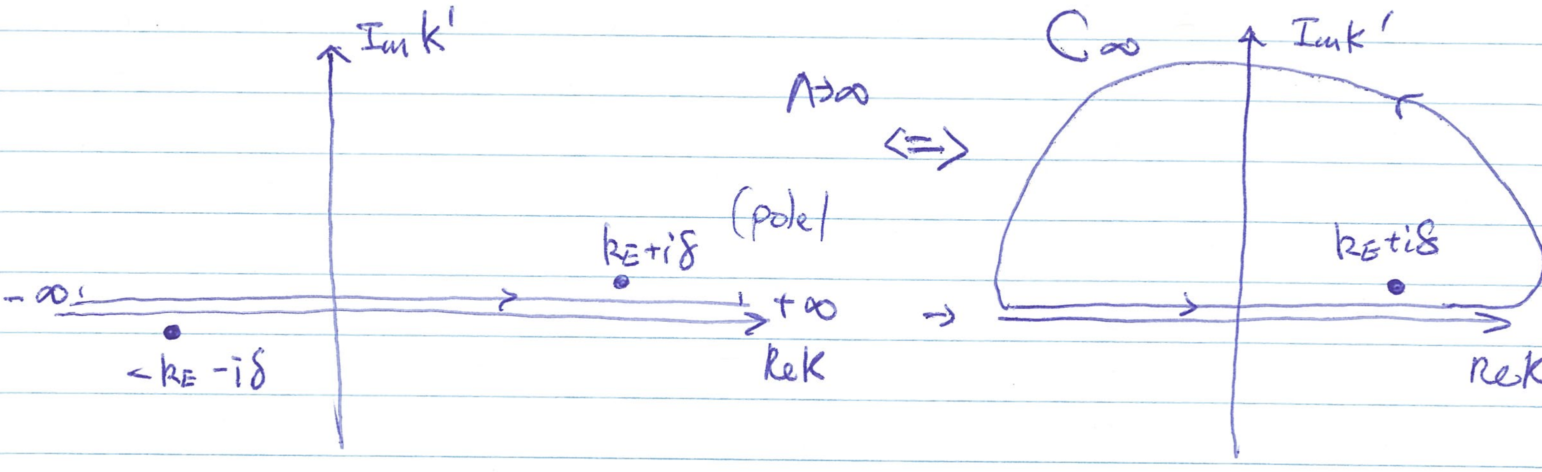
$$I(E) = \int_\Lambda \frac{d^3k'}{(2\pi)^3} \frac{2}{k'^2 - k_E^2 - i\delta}$$

$$= \int_\Lambda \frac{d^3k'}{(2\pi)^3} \left[ \frac{2}{k'^2 - k_E^2 - i\delta} - \frac{2}{k'^2} \right] + \underbrace{\int_\Lambda \frac{d^3k'}{(2\pi)^3} \frac{2}{k'^2}}_{\text{Regularization term}}$$

$$\overset{\Lambda\to\infty}{\cong} \frac{k_E^2}{2\pi^2} \overbrace{\int_{-\infty}^{+\infty} dk' \frac{1}{k'^2 - k_E^2 - i\delta}}^{\oint_{C_\infty}} + \frac{1}{\pi^2}\Lambda$$

$$\overset{\Lambda\to\infty}{\cong} \frac{k_E^2}{2\pi^2}\, 2\pi i \,\mathrm{Res}\left(\frac{1}{k'^2 - k_E^2 - i\delta}\right)\Bigg|_{k' = k_E + i\delta} + \frac{1}{\pi^2}\Lambda$$

$$\cong \frac{i k_E}{2\pi} + \frac{1}{\pi^2}\Lambda$$




$$\frac{1}{k'^2 - k_E^2 - i\delta}$$ has poles at $k = \pm(k_E + i\delta)$