Supplementary Material 3

1)

$$T_{\vec{k}_f, \vec{k}_i}(E) = V_{\vec{k}_f, \vec{k}_i} + \frac{1}{\Omega} \sum_{\vec{k}'} \frac{V_{\vec{k}_f, \vec{k}'} T_{\vec{k}', \vec{k}_i}(E)}{E - \frac{k'^2}{2} + i\delta}$$

$$V_{\vec{k}_f, \vec{k}_i} = \Omega \langle \vec{k}_f | V | \vec{k}_i \rangle, \quad T_{\vec{k}_f, \vec{k}_i}(E) = \Omega \langle \vec{k}_f | V | \psi^{TOT}_{\vec{k}_i} \rangle$$

$$\langle \vec{r} | \psi^{TOT}_{\vec{k}_i} \rangle \overset{r \to \infty}{=} \langle \vec{r} | \vec{k}_i \rangle \quad \text{(boundary condition)}$$

$$= \frac{1}{\sqrt{\Omega}} e^{i \vec{k}_i \cdot \vec{r}}$$

$\to$ Normalization factor ($\frac{1}{\sqrt{\Omega}}$)

2) pseudo potential (approximates a short range potential

$$V_{\vec{k}_f, \vec{k}_i} = \begin{cases} g, & |\vec{k}_f|, |\vec{k}_i| \leq \Lambda \\ 0, & \text{otherwise.} \end{cases}$$

($g \to$ strength)

$g = \int V(\vec{r}\,') d^3\vec{r}\,'$ — interaction strength

$\Lambda \sim \frac{1}{R_0}$ moment cut-off to reflect the range of potential.

def. $a_s = - \lim_{k \to 0} \frac{\sin \delta(k)}{k}$