Jan 8, 2019

HW Set 0 Solutions

$$j_\ell(\rho) = \rho^\ell \left[-\frac{1}{\rho}\frac{\partial}{\partial \rho}\right]^\ell j_0(\rho), \quad n_\ell(\rho) = \rho^\ell \left(-\frac{1}{\rho}\frac{\partial}{\partial\rho}\right]^\ell n_0(\rho)$$

A) $\rho \to 0$, $j_0(\rho) = \frac{\sin\rho}{\rho} = \sum_{n=0}^{\infty} \frac{(-1)^n \rho^{2n}}{(2n+1)!}$, $n_0(\rho) = -\frac{\cos\rho}{\rho}$

$$j_\ell(\rho) = \rho^\ell \left[-\frac{1}{\rho}\frac{\partial}{\partial\rho}\right]^\ell j_0(\rho) = \sum_{n \geq \ell} (-1)^n \frac{\rho^{2n-2\ell+\ell}}{(2n+1)!} (2n)!!$$

$$\stackrel{\rho\to 0}{=} (-1)^\ell \frac{\rho^\ell}{(2n+1)!!} + O(\rho^{\ell+1}) + \cdots, \quad (2n+1)!! = (2n+1)(2n-1)\cdots 1$$

$$n_\ell(\rho) = \rho^\ell \left[-\frac{1}{\rho}\frac{\partial}{\partial\rho}\right]^\ell \left\{-\sum_{n=0}^{\infty} (-1)^n \frac{\rho^{2n-1}}{2n!}\right\}$$ ← Most singular term $n=0$

$$\stackrel{\rho\to 0}{=} (-1)^{\ell+1} \frac{1}{\rho^{\ell+1}} (2\ell-1)!! + \text{Other less singular terms}$$

B) $h_0^{(1)}(\rho) = (-i)\frac{e^{i\rho}}{\rho}$ $\quad h_0^{(2)}(\rho) = i\frac{e^{-i\rho}}{\rho}$

$\rho \to \infty$, $h_{0\ell}^{(1)}(\rho) = \rho^\ell \left[-\frac{1}{\rho}\frac{\partial}{\partial\rho}\right]^\ell h_0^{(1)}(\rho) \stackrel{\rho\to\infty}{\Rightarrow} (-i)^{\ell+1} \frac{e^{i\rho}}{\rho}$ + other therms..

$$h_{0\ell}^{(2)}(\rho) \stackrel{\rho\to\infty}{\Rightarrow} (i)^{\ell+1} \frac{e^{-i\rho}}{\rho}$$

$$j_\ell(\rho) \xrightarrow{\rho\to\infty} \frac{1}{2}\left[h_\ell^{(1)} + h_\ell^{(2)}\right] = \frac{\sin(\rho - \frac{\ell\pi}{2})}{\rho}, \quad n_\ell(\rho) \Rightarrow \frac{\cos(\rho - \frac{\ell\pi}{2})}{\rho}$$