

Jan 8, 2019

HW Set 0 Solutions

$$j_l(p) = p^l \left[-\frac{1}{p} \frac{\partial}{\partial p} \right]^l j_0(p), \quad n_l(p) = p^l \left[-\frac{1}{p} \frac{\partial}{\partial p} \right]^l n_0(p)$$

A) $p \rightarrow 0, \quad j_0(p) = \frac{\sin p}{p} = \sum_{n=0}^{\infty} \frac{(-1)^n p^{2n}}{(2n+1)!}, \quad n_0(p) = -\frac{\cos p}{p}$

$$j_l(p) = p^l \left[-\frac{1}{p} \frac{\partial}{\partial p} \right]^l j_0(p) = \sum_{n \geq l} \frac{(-1)^n p^{2n-2l+l}}{(2n+1)!} (2n)!!$$

$$\stackrel{p \rightarrow 0}{=} (-1)^l \frac{p^l}{(2l+1)!!} + O(p^{l+1}), \quad (2n+1)!! = (2n+1)(2n-1)\dots 1$$

$$n_l(p) = p^l \left[-\frac{1}{p} \frac{\partial}{\partial p} \right]^l \left\{ -\sum_{n=0}^{\infty} \frac{(-1)^n p^{2n-1}}{2n!} \right\} \leftarrow \text{Most singular term } n=0$$

$$\stackrel{p \rightarrow 0}{=} (-1)^{l+1} \frac{1}{p^{l+1}} (2l-1)!! + \text{Other less singular terms}$$

B) $h_0^{(1)}(p) = (-i) \frac{e^{ip}}{p}, \quad h_0^{(2)}(p) = i \frac{e^{-ip}}{p}$

$$p \rightarrow \infty, \quad h_{0l}^{(1)}(p) = p^l \left[-\frac{1}{p} \frac{\partial}{\partial p} \right]^l h_0^{(1)}(p) \stackrel{p \rightarrow \infty}{\Rightarrow} (-i)^{l+1} \frac{e^{ip}}{p} + \text{other terms}$$

$$h_{0l}^{(2)}(p) \stackrel{p \rightarrow \infty}{\Rightarrow} (i)^{l+1} \frac{e^{-ip}}{p}$$

$$j_l(p) \stackrel{p \rightarrow \infty}{\Rightarrow} \frac{1}{2} [h_{0l}^{(1)} + h_{0l}^{(2)}] = \frac{\sin(p - \frac{l\pi}{2})}{p}, \quad n_l(p) \Rightarrow \frac{\cos(p - \frac{l\pi}{2})}{p}$$