

Phys 501: Quantum Mechanics II

**Homework Set IX (Due 1230pm, Friday, April 5, 2019)**

Review path integrals in Euclidean space, application to the classical forbidden regime.

Prob.1. Derive the *instanton* solution (named by t'Hooft) for the double well problem for both positive tau and negative tau. How much does the asymptotic behavior at large positive or large negative tau depend on the details of the model? i.e. what is the general condition under which the solution always has the "instant" character? (See textbook discussions in page 619 as well.) Think about most general potentials with two degenerate minima in potential. Will that be enough for the instanton to be "instant"?

Prob.2 Compute the action of the classical solution and get estimate of the tunneling matrix element.

Prob.3 (Bonus problem) When one takes into the fluctuations, one shall expect that paths spread over certain spatial region surrounding the classical solution. If you follow the suggestion in the textbook and compute the Gaussian fluctuations around the classical solution, Can you find out what is the quantum spread around the classical solution? (Yes, I haven't talked about it but some of you might be able to crack out the answer if you know which quantity you need compute to answer this question.