Phys 501: Quantum Mechanics II

Homework Set VII (Due 1230pm, Monday, March 6, 2019)

Review Dirac algebra, Dirac equation in the massless limit.
Non-relativistic limit stuff.

Prob. 1 The Dirac equation can be written in terms of standard gamma matrices. [See Dirac matrices in the attached note (given to you during my lecture). For more discussions, see also Eq. 3.25 and Eq. 3.31 in *An introduction to QFT by Peskin and Schroede*r.] Show that it is equivalent to the alpha-\beta representation. Hint: what kind of unitary transformation that connects these two representations? The transformation is similar to what I did for Weyl representation during the lecture but not identical.

Prob. 2 An arbitrary Dirac state can be splitted into two parts via the projection operators P_{\pm} (see attached, defined by gamma_5 matrices). Show in details that (this problem overlaps with my lecture but fill in the gaps.)

1) Each projected state is the eigen state of the its projection operator and they are orthogonal for arbitrary Dirac states.
2) Projected states satisfy one of the Weyl fermion equations and hence the projection operators do exactly what they are supposed to do to project out right-handed and left –handed Weyl fermions. Show explicitly the two conservation laws discussed during the lectures.

Prob.3 In the massless limit, we have shown in additional to the charge conservation, gamma-5 current is also conserved. In the massive limit, derive a similar "continuity equation" for gamma-5 current and show that it is not conserved explicitly.

Prob.4. Consider a massive Dirac particle. Assume the state is initially prepared in a plane wave state with helicity equal to +1 or in a Right handed Weyl state. Compute the Gamma-5 charge as a function of time.