Phys 501: Quantum Mechanics II

Homework Set V (Due 1230pm, Wed, Feb 27, 2019)

Review Dirac algebra, Dirac equations and Dirac spectrum (chapter 20)

Prob. 1 Show explicitly that alpha, beta matrices we introduced during the last lecture (see also text book Eq.(20.1.12)) indeed form a representation of the Dirac Algebra and satisfy the algebraic relations (see also Eq.20.1.11).

Prob.2 For a momentum pointing along a direction specified by a unit vector \hat{p}, find the solutions to Dirac equation as a function of momentum only (magnitude and direction).
Note: There are equivalent ways to write the solutions depending your choices of 2-component spinors \chi. Choose to work with 2-component spinors which describe spins pointing along the direction of momentum or opposite to the momentum. In this way, spinors \chi is identical to spinor \phi (up to a prefactor). (See supplementary stuff.)

Prob.3. Exercise 20.1.1 in the text book. Prove the continuity.

Prob.4. Take the current density expression in Prob.2, compute the current density carried by a plane wave state (4-component Dirac spinor or bi-spinor).

a) In the non-relativistic limit, how does the current density look like?
b) For states with a given momentum (4 states all together, 2 with positive energies, 2 negative ones), do they all carry the same current density?
c) Does the current density depend on the choices of 2-component spinor wave function \chi or \phi ?
d) Now look at the current density in 2 positive energy states. Compared with the current density in 2 negative energy states. Are they the same?

[Again there are many equivalent ways of writing \chi, \phi, you might compute the density with a particular form of \chi and \phi. But can you prove your conclusion is general and independent of choices \chi and \phi?]