Phys 501; Quantum Mechanics II

Homework Set IV (Due 1230pm, Wednesday, Feb. 13, 2019)

Understand the relation between the phases in T-matrix (complex function of E), phase shifts in scattering waves and scattering amplitude f(\theta).

Prob. 1

Find out the phase shifts for l=0,1 partial waves for Yukawa potential (see textbook page 531) in the Born approximation. Compare the two amplitudes.

Prob.2. Take the k-space pseudo potential we introduced during the last lecture as the starting point for short range interactions. Restrict to S-waves. Some of the conclusions you have had in previous HW sets. Here is to apply the T-matrix approach to address the similar issues.

1) Assume the interaction is repulsive. Find the scattering length a_s as a function of the potential strength g and show that for a fixed range, the scattering length saturates as the repulsion becomes infinite.

2) Show the phase shift is always much less than \pi/2 for arbitrary repulsive interactions if the range is much smaller than the incident wave lengths.

3) Show that is not the case for attractive interaction. Derive the condition for resonance (in terms of g and cut-off lambda), i.e. when the phase shift is exactly \pi/2 for arbitrary incident wave lengths. Hint: T matrix contains all the information you want to have about the phase shifts.

4) Find the scattering length a_s in terms of g and lambda. How good is this characterization for a finite range potential? i.e. estimate the leading order correction due to finite lambda for fixed scattering length a_s. (this requires one step further than what I did during my lectures.) Hint: recall the definition of the scattering length a_s.

5) Show a_s indeed is equal to a_B in the presence of bound states. (Hint: Use the analytical continuation from the positive energy to the negative energy. For this pseudo potential with attraction, the T-matrix has a pole along the negative energy axis within the cut off energy; this pole can be identified as the bound state energy.)