Phys 501; Quantum Mechanics II

Homework Set I (Due 1230pm, Monday, Jan 16, 2019)

Objectives:
Understand the basic notions and ideas in scattering theory:

1) *Characterization of scattered waves: scattering amplitude, lth partial wave amplitude and Scattering cross-sections (differential and total);*
2) *Characterization of scattering states by phase shifts and the relation between phase shifts and scattering amplitudes;*
3) *Computing phase shifts and scattering cross sections.*

Math requirements:
** You need to know how to work with spherical Bessel functions (first kind and second kind), Henkel functions etc.*
*** Understand the asymptotic behaviors of these functions at large or small distances.*

Prob. 1 Scattering by a short range potential

Large numbers of scattering problems involve short range potentials such as Yukawa potential in nuclear physics, Van der Waals potentials between distant atoms and chemical valence bond interactions at atomic scales. Computation of scattering due to any realistic potentials such as scattering between atoms or nucleons usually is very involved and requires enormous computation power. Fortunately, sophisticated renormalization arguments show that for certain subclass of scattering problems (i.e. the renormalizable ones which can be characterized by an infinite number of identical theories defined at all different energy scales; I believe this was first pointed out in 80s.) and if one is interested in the long wave length limit, then there is a vast energy window where scattering by a complicated, realistic short range potential can be described by a simple pseudo potential, a math model which can be handled without a computer. This is what you need to work out here, scattering by a square well potential as a simple pseudo potential. Many conclusions you derived here are universal.

1) Show if the incoming wave is a *spherical wave* with a given angular momentum l, the outgoing wave is a spherical wave with amplitude c but |c|=1. Therefore, c is fully characterized by a phase only. That phase is defined as 2\delta_l, with \delta_l being the phase shift of lth partial wave. Hint: If |c| is not equal to one, show that spatial probability density is time dependent.

2) For the incoming plane wave, find the general relation between the phase shifts for the l=0 or s-wave channel and the potential depth and range of interaction.

3) You shall be able to get simple results when the Born parameter **B** is very small. What is the cross section in this limit? And how does it depend on the energy of the incident particles [in this part as well as the rest of the problem, the incident wave energy is much smaller than the depth of the interaction potential. In this limit, your result shall have a very simple form.]

4) Identify at which value of the potential depth (for a fixed range) or the Born parameter **B**, the scattering cross section becomes infinite when k approaches zero.

5) Find out when the scattering cross sections are infinite, what happens to bound states below the continuum. You are looking at so called the resonance phenomena. Hint: this is a standard eigen value problem (effectively 1D).

6) At the resonance, how the scattering cross section looks like as a function of k, incident wave vector or energy. Assume the energy is very low. Compared to the depth of potential etc.

Beyond S-waves

7) Show that generically, l=1,2,3,… high partial wave scattering phase shifts are much smaller compared to s-wave ones and are negligible when the incident wave length is long. Speculate when the higher partial waves become important for a given potential range.

8) Most ultra cold atoms scattering studied in labs take place at wave lengths of order of a few hundred nano meters. Given what you already know about atoms, do you think S-wave scattering is dominating in that limit, or not. Hint: *if you decide to use a square-well potential to approximate atomic interactions, what is your estimate of the parameters for the potential well ? This part doesn't require a lot of computation but need to understand how to connect what you have obtained in 1-7) to atomic scattering one can study in labs. A simple application of what you have learned.*