

The Ammonia Maser

9-1 The states of an ammonia molecule

In this chapter we are going to discuss the application of quantum mechanics to a practical device, the ammonia maser. You may wonder why we stop our formal development of quantum mechanics to do a special problem, but you will find that many of the features of this special problem are quite common in the general theory of quantum mechanics, and you will learn a great deal by considering this one problem in detail. The ammonia maser is a device for generating electromagnetic waves, whose operation is based on the properties of the ammonia molecule which we discussed briefly in the last chapter. We begin by summarizing what we found there.

The ammonia molecule has many states, but we are considering it as a two-state system, thinking now only about what happens when the molecule is in any specific state of rotation or translation. A physical model for the two states can be visualized as follows. If the ammonia molecule is considered to be rotating about an axis passing through the nitrogen atom and perpendicular to the plane of the hydrogen atoms, as shown in Fig. 9-1, there are still two possible conditions—the nitrogen may be on one side of the plane of hydrogen atoms or on the other. We call these two states $|1\rangle$ and $|2\rangle$. They are taken as a set of base states for our analysis of the behavior of the ammonia molecule.

9-1 The states of an ammonia molecule

9-2 The molecule in a static electric field

9-3 Transitions in a time-dependent field

9-4 Transitions at resonance

9-5 Transitions off resonance

9-6 The absorption of light

MASER = Microwave Amplification by Stimulated Emission of Radiation

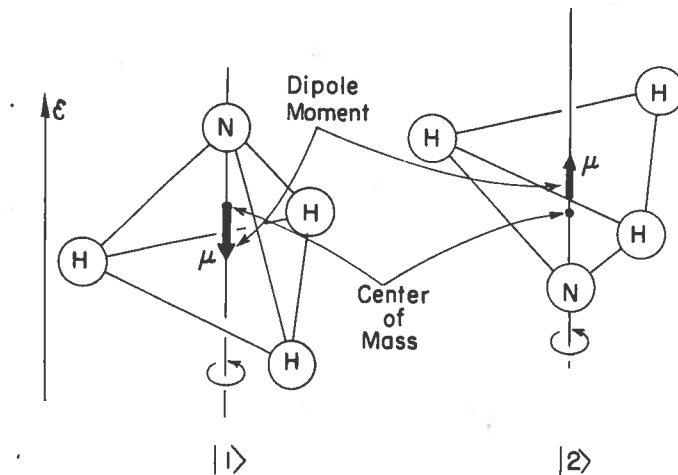


Fig. 9-1. A physical model of two base states for the ammonia molecule. These states have the electric dipole moments μ .

In a system with two base states, any state $|\psi\rangle$ of the system can always be described as a linear combination of the two base states; that is, there is a certain amplitude C_1 to be in one base state and an amplitude C_2 to be in the other. We can write its state vector as

$$|\psi\rangle = |1\rangle C_1 + |2\rangle C_2, \quad (9.1)$$

where

$$C_1 = \langle 1 | \psi \rangle \quad \text{and} \quad C_2 = \langle 2 | \psi \rangle.$$

These two amplitudes change with time according to the Hamiltonian equations, Eq. (8.43). Making use of the symmetry of the two states of the ammonia molecule, we set $H_{11} = H_{22} = E_0$, and $H_{12} = H_{21} = -A$, and get the

region in which there is a large transverse electric field. The electrodes to produce the field are shaped so that the electric field varies rapidly across the beam. Then the square of the electric field $\mathcal{E} \cdot \mathcal{E}$ will have a large gradient perpendicular to the beam. Now a molecule in state $|I\rangle$ has an energy which increases with \mathcal{E}^2 , and therefore this part of the beam will be deflected toward the region of lower \mathcal{E}^2 . A molecule in state $|II\rangle$ will, on the other hand, be deflected toward the region of larger \mathcal{E}^2 , since its energy decreases as \mathcal{E}^2 increases.

Incidentally, with the electric fields which can be generated in the laboratory, the energy $\mu\mathcal{E}$ is always much smaller than A . In such cases, the square root in Eqs. (9.30) can be approximated by

$$A \left(1 + \frac{1}{2} \frac{\mu^2 \mathcal{E}^2}{A^2} \right). \quad (9.32)$$

So the energy levels are, for all practical purposes,

$$E_I = E_0 + A + \frac{\mu^2 \mathcal{E}^2}{2A} \quad (9.33)$$

and

$$E_{II} = E_0 - A - \frac{\mu^2 \mathcal{E}^2}{2A}. \quad (9.34)$$

And the energies vary approximately linearly with \mathcal{E}^2 . The force on the molecules is then

$$F = \frac{\mu^2}{2A} \nabla \mathcal{E}^2. \quad (9.35)$$

Many molecules have an energy in an electric field which is proportional to \mathcal{E}^2 . The coefficient is the polarizability of the molecule. Ammonia has an unusually high polarizability because of the small value of A in the denominator. Thus, ammonia molecules are unusually sensitive to an electric field. (What would you expect for the dielectric coefficient of NH_3 gas?)

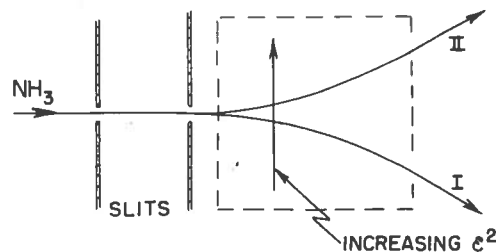


Fig. 9-3. The ammonia beam may be separated by an electric field in which \mathcal{E}^2 has a gradient perpendicular to the beam.

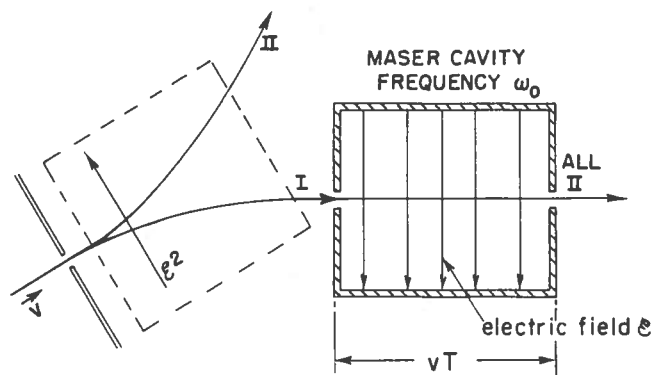


Fig. 9-4. Schematic diagram of the ammonia maser.

9-3 Transitions in a time-dependent field

In the ammonia maser, the beam with molecules in the state $|I\rangle$ and with the energy E_I is sent through a resonant cavity, as shown in Fig. 9-4. The other beam is discarded. Inside the cavity, there will be a time-varying electric field, so the next problem we must discuss is the behavior of a molecule in an electric field that varies with time. We have a completely different kind of a problem—one with a time-varying Hamiltonian. Since H_{ij} depends upon \mathcal{E} , the H_{ij} vary with time, and we must determine the behavior of the system in this circumstance.

To begin with, we write down the equations to be solved:

$$ih \frac{dC_1}{dt} = (E_0 + \mu\mathcal{E})C_1 - AC_2, \quad (9.36)$$

$$ih \frac{dC_2}{dt} = -AC_1 + (E_0 - \mu\mathcal{E})C_2.$$