Solutions to the midterm Prob.

Prob. I.

$$\psi_n^{(0)}(x) = \sqrt{\frac{2}{L}}\,\sin\left(\frac{n\pi x}{L}\right), \quad E_n^{(0)} = \frac{\pi^2}{2mL^2}\,n^2, = \varepsilon_0 n^2,$$

$$n = 1, 2, \ldots$$

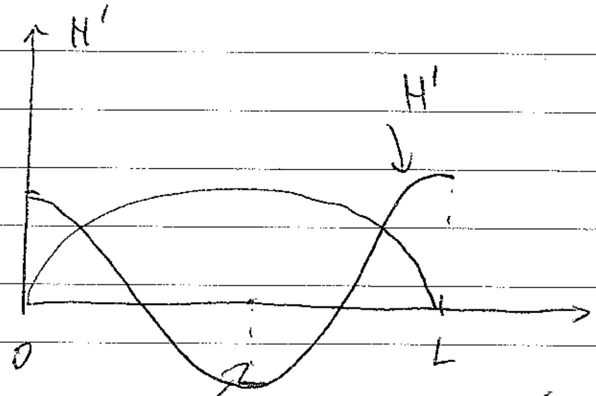
a)

$$\underbrace{\langle n \,|\, H' \,|\, n'=1 \rangle}_{\text{ground state}} = \frac{2}{L}\int_0^L \sin\frac{n\pi x}{L}\; A\; \underbrace{\cos\frac{2\pi x}{L}\,\sin\frac{\pi x}{L}}_{\frac{1}{2}\left(\sin\frac{3\pi x}{L} - \sin\frac{\pi x}{L}\right)}\, dx$$

$$= \frac{A}{2}\left(\frac{2}{L}\int_0^L \sin\frac{n\pi x}{L}\left(\sin\frac{3\pi x}{L} - \sin\frac{\pi x}{L}\right) dx\right)$$

$$= \frac{A}{2}\left(\delta_{n,3} - \delta_{n,1}\right) \quad (n \neq 1)$$

$$E^{(1)}_{n=1} = \langle 1 | H' | 1 \rangle = \boxed{-}\frac{A}{2}$$ negative sign



negative at $x = L/2$

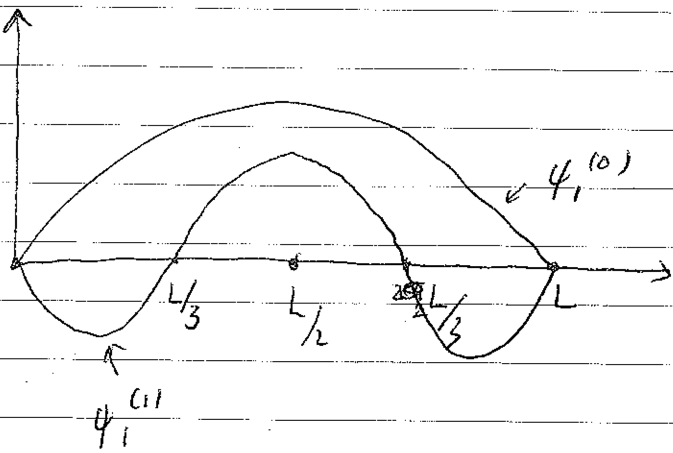
$$E^{(2)}_{n=1} = -\sum_{n'} \frac{\langle n' | H' | 1 \rangle \langle 1 | H' | n' \rangle}{E_{n'}^{(0)} - E_1^{(0)}}$$ (only $n' = 3$ term nonvanishing)

$$= -\frac{1}{(9-1)\,\varepsilon_0}\cdot\left(\frac{A}{2}\right)^2 = \boxed{-\frac{A^2}{32\varepsilon_0}}$$

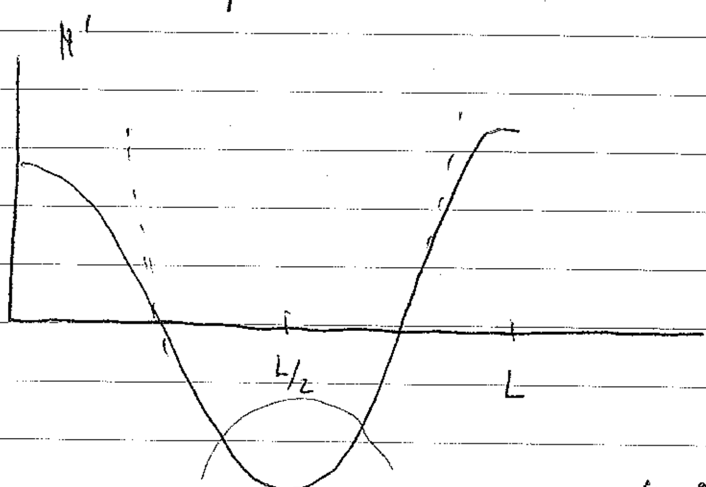
b)

$$\psi_1^{(1)} = \sum_{n' \neq 1} C_{n'}^{(1)}\,\psi_{n'}^{(0)}, \qquad C_{n'}^{(1)} = -\frac{\langle n' | H' | 1 \rangle}{E_{n'}^{(0)} - E_1^{(0)}} = -\frac{A}{16\varepsilon_0}\,\delta_{n',3}$$

$$\psi_1^{(1)} = -\frac{A}{16\varepsilon_0}\left(\sqrt{\frac{2}{L}}\sin\frac{3\pi x}{L}\right)$$



c) A is very large



approximate the potential as a H.O.

$$H'_{eff} = \frac{1}{2}m\omega^2\left(x-\frac{L}{2}\right)^2, \qquad m\omega^2 = A\,\frac{4\pi^2}{L^2}$$

$$l_{HO} = \frac{1}{\sqrt{m\omega}} = \frac{L^{1/2}}{(mA)^{1/4}} \qquad A\to\infty,\ \underbrace{l_{HO} \sim \frac{1}{A^{1/4}}}$$

localized near $x = L/2$

Prob. 2

a)

$j_z = -\frac{1}{2}$.

$$|j=\tfrac{3}{2}, j_z=-\tfrac{1}{2}\rangle = \frac{1}{\sqrt{3}}\left(|+\rangle\otimes|1,-1\rangle + \sqrt{2}|-\rangle\otimes|1,0\rangle\right), \quad \mathcal{E}_{3/2} = \frac{A}{2}$$

$$|j=\tfrac{1}{2}, j_z=-\tfrac{1}{2}\rangle = \frac{1}{\sqrt{3}}\left(-\sqrt{2}\,|+\rangle\otimes|1,-1\rangle + |-\rangle\otimes|1,0\rangle\right) \quad \mathcal{E}_{1/2} = -A$$

$$\Psi(t=0) = |-\rangle\otimes|1,0\rangle$$

$$= \frac{1}{\sqrt{3}}\left(\sqrt{2}\,|\tfrac{3}{2},-\tfrac{1}{2}\rangle + |\tfrac{1}{2},-\tfrac{1}{2}\rangle\right)$$

$$\Psi(t=T) = \frac{1}{\sqrt{3}}\Big(\sqrt{2}\cdot\left(\frac{1}{\sqrt{3}}\right)\left(|+\rangle\otimes|1,-1\rangle + \sqrt{2}|-\rangle\otimes|1,0\rangle\right)e^{-i\frac{At}{2}}$$

$$+ \frac{1}{\sqrt{3}}\left(-\sqrt{2}\,|+\rangle\otimes|1,-1\rangle + |-\rangle\otimes|1,0\rangle\right)e^{iAt}\Big)$$

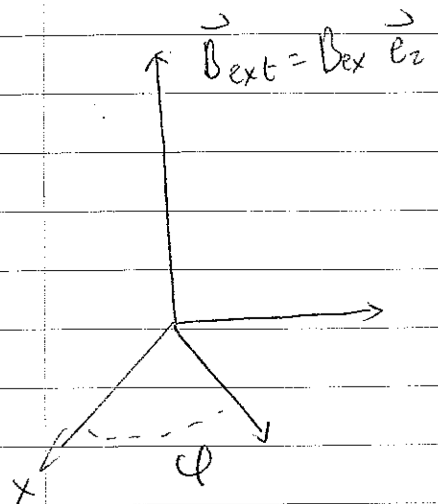
$$P_+ = \frac{1}{9}\cdot 2\left(2 - 2\cos\frac{3At}{2}\right) = \frac{4}{9}\left(1-\cos\frac{3At}{2}\right)$$

$$P_- = \frac{1}{9}\left(5 + 4\cos\frac{3At}{2}\right)$$

$P_+ + P_- = 1$ Conserved

b) When B is very strong, S_z, L_z are good quantum numbers and $|\pm\rangle\otimes|1,0\rangle$ is almost a steady state; the Probability of finding $|+\rangle$ when initially at $|-\rangle$ is almost zero.

c)



For very lage B-field, $|1, l_z=0\rangle$ is an eigenstate, or ~~stationary~~ steady state.

One only needs to study the Spin.

Spin precesses in the xy-plane in this case.

$$H_{Zeeman} = \frac{eB}{2m} S_z = \gamma S_z$$

$$|\psi(t)\rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} e^{-i\gamma\frac{t}{2}} \\ e^{i\gamma\frac{t}{2}} \end{pmatrix} \otimes \underbrace{|l=1, l_z=0\rangle}_{\text{no dynamics}}$$

$P_+ = P_- = \frac{1}{2}$ no oscillations.

precession angle is $\varphi(t) = \underline{\gamma} t$ $\quad \gamma$ - Larmor frequency.