Phys 402 Midterm Exam, March 1, 930-1050am, Hebb 12

You can bring two formula sheets. No books and other references. No internet connections. Manage your time wisely. Show all necessary steps that lead to your conclusion. Please write your name and student ID clearly on the cover page of exam booklet.

Total 100pts

Prob. I (50pts)

Consider a particle moving in a one-dimensional system of size L with a periodical boundary condition, i.e. $\psi(x) = \psi(x + L)$. A small perturbation of the following form is applied,

$$H^{'} = A\cos\frac{6\pi x}{L} \tag{1}$$

where $A(> 0)$ is the amplitude of perturbation.

a)(15) Find the eigen states and eigen values for this free particle with the periodical boundary condition, in the absence of perturbation $H^{'}$.

b)(10) Find the first order correction to the ground state energy due to the perturbation.

c)(25) Find the second order correction to the ground state energy.

Prob. II (50pts)

Because of an anisotropy along the z direction, the spin-orbit coupling of an electron in an atom in a crystal-X takes the following form instead

$$H_{SO} = A(\mathbf{S_x}\cdot\mathbf{L_x} + \mathbf{S_y}\cdot\mathbf{L_y}) \tag{2}$$

where $\mathbf{S}$ is the electron spin and $\mathbf{L}$ is the angular momentum. A is a constant. (Assume this is the only effect of the crystal on the electronic states.)

a) (30) What are the eigen states and eigen values of this Hamiltonian for the p-orbitals or $l = 1$ states in the atom ?

b) (10) Are J_z, J^2 both good quantum numbers in this case?

c) (10) Now an electron is initially in a product state $|S = 1/2, S_z = \frac{1}{2} > \otimes |l = 1, l_z = 1 >$ with spin pointing along the z direction. What are the probabilities of finding the electron with $S_z = -\frac{1}{2}$ at a later time t?