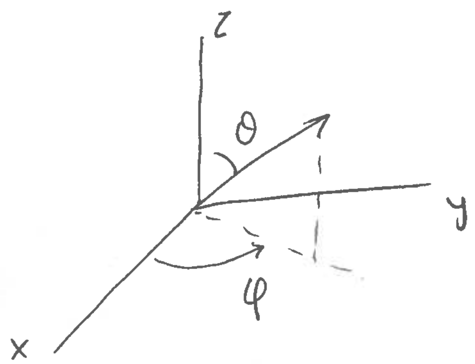
Home work III A

1) Bloch Sphere State $|\hat{\Omega}\rangle$ is defined as

$$\vec{S}\cdot\hat{\Omega}\,|\hat{\Omega}\rangle = \frac{\hbar}{2}\,|\hat{\Omega}\rangle .$$

Following the previous HW set, $|\hat{\Omega}\rangle = \begin{pmatrix} \cos\frac{\theta}{2} \\ \sin\frac{\theta}{2}\, e^{i\varphi} \end{pmatrix}$



$$\langle S_x\rangle = \frac{\hbar}{2}\sin\theta\cos\varphi$$

$$\langle S_y\rangle = \frac{\hbar}{2}\sin\theta\sin\varphi$$

$$\langle S_z\rangle = \frac{\hbar}{2}\cos\theta$$

$\langle\vec{S}\rangle = \frac{\hbar}{2}\hat{\Omega}$ orenitation along the direction of $\hat{\Omega}$

2) $|t=0\rangle = \begin{pmatrix} \cos\theta_0/2 \\ \sin\theta_0/2\, e^{i\varphi_0} \end{pmatrix}$ $\quad \hat{n}_0 = \hat{n}_0(\theta_0, \varphi_0)$

polar angle for $\hat{n}_0$.

$$|t\rangle = \begin{pmatrix} \cos\frac{\theta_0}{2}\, e^{-i\frac{B\gamma t}{2}} \\ \sin\frac{\theta_0}{2}\, e^{i\varphi_0}\cdot e^{i\frac{B\gamma t}{2}} \end{pmatrix},$$

(Assume $H = \frac{\gamma}{2}\vec{B}\cdot\vec{S}$)

$= \frac{\gamma B}{2} S_z$

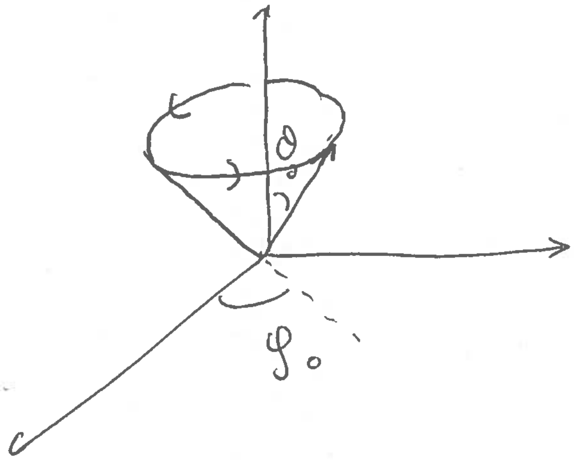
$\gamma = g\frac{e}{m} > 0$

(differs from textbook def.)

$$= e^{-i\frac{B\gamma t}{2}} \begin{pmatrix} \cos\frac{\theta_0}{2} \\ \sin\frac{\theta_0}{2}\, e^{i\varphi_0 + B\gamma t} \end{pmatrix}$$

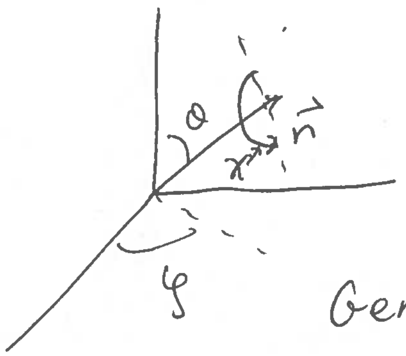
$\hat{n}(t) = \hat{n}(\theta(t), \varphi(t))$, $\quad \theta(t) = \theta_0$, $\quad \varphi(t) = \varphi_0 + B\gamma t$

"Right hand Rule" when it is plus.



"Larmor Precession"

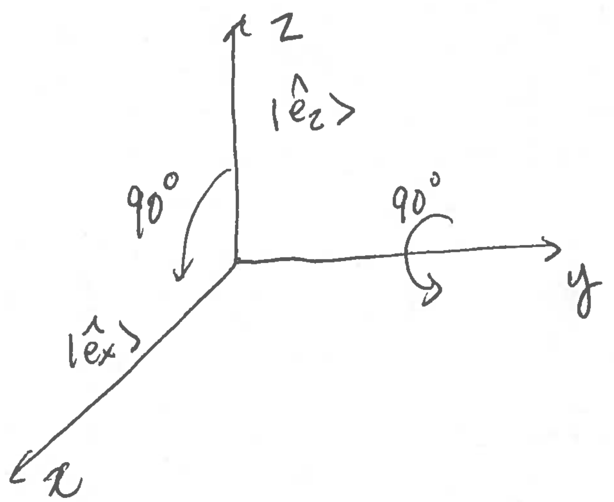
3) $$U(\hat{n}, \chi) = \cos\frac{\chi}{2} - i\sin\frac{\chi}{2}\,\vec{\sigma}\cdot\hat{n} = e^{-i\frac{\vec{\sigma}\cdot\hat{n}}{2}\chi}$$



Generally,

$$|\hat{\Omega}'\rangle = U(\hat{n}, \chi)\,|\hat{\Omega}\rangle$$

After Rotation $\quad$ Before Rotation



$$|\hat{\Omega}\rangle = |\hat{e}_z\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$\hat{n} = \hat{e}_y, \quad \chi = \frac{\pi}{2}$$

$$U(\hat{e}_y, \tfrac{\pi}{2}) = \frac{1}{\sqrt{2}}(1 - i\sigma_y)$$

$$= \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix}$$

$$|\hat{\Omega}'\rangle = U(\hat{e}_y, \tfrac{\pi}{2})\,|\hat{e}_z\rangle$$

$$= \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ 1 \end{pmatrix} = |\hat{e}_x\rangle$$

desired state

4) in General, one can generate SO(3) Rotations Matrix using SU(2) Rotations (first pointed out by Hamilton when studying Classical motion of a top)

$$R_{\alpha\beta}(\hat{n}, \chi) = \frac{1}{2} tr\left(U^{+} \sigma_\alpha U \sigma_\beta\right) \quad (*)$$

SO(3)

$U = U(\hat{n}, \chi)$ — SU(2) Rotation introduced before

$\alpha, \beta = x, y, z$

$R_{\alpha\beta}(\hat{n}, \chi)$ is a 3x3 matrix

For SO(3) Rotation, $r'_\alpha = R_{\alpha\beta} r_\beta$, $\alpha, \beta = x, y, z$

or

$$\vec{r}\,' = R\,\vec{r} \quad (**)$$

$\vec{r}\,'$ — After Rotation

$\vec{r}$ — vector before Rotation

R – SO(3) Rotation Matrix can be defined in (*)

if you can show R in (**) can be generated by R defined in (*), please come to my office

—— I'll take you out for a coffee!