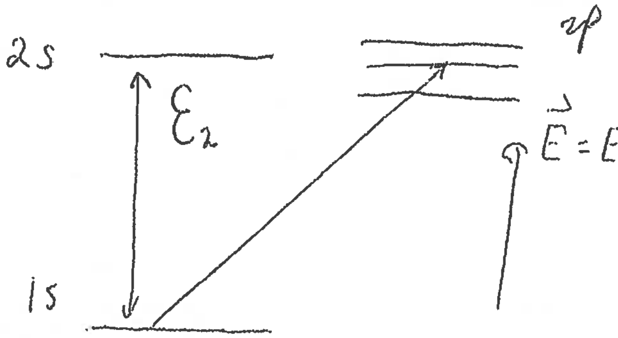
Phys 402: Applications of Quantum Mechanics

Homework 10 (Total 2 problems; due 930am, Thursday, April 7, 2016)
[To receive full credits, please show all necessary steps that lead to your answers.]

1) John wants to optically pump an electron from 1s to one of the 2p orbital in a hydrogen atom so that at the end at certain moment the electron occupies the 2p orbital with nearly 100% (or more than 90%) probability. The maximum electric field in the lasers available in John's lab is around 1000V/m. Estimate what range of the laser frequencies John needs to work with to achieve the above goal.

2) Consider a hydrogen atom at 2p state. Estimate the life time of the atom due to the spontaneous emission. (Pre-read Section 9.2.3 and 9.3.1 before starting. This is a simple example where you can apply what you have learned so far. You might need a little bit information from Chapter 5, section 5.4.5 on the black-body spectrum to understand the spontaneous emission.)

Prob. 3.



$\vec{E} = E_z \cos\Omega t \, \hat{e}_z$

$$H' = -\vec{p}\cdot\vec{E} = e\,z\,E_z \cos\Omega t$$

$$\langle f|H'|i\rangle = \underbrace{\langle 2p|z|1s\rangle}_{\sim a_{Bohr}} e E_z \cos\Omega t$$

In the form of perturbations, $H = H_0 + H' = H_0 + H_{fi}\cos\Omega t$

$$C_{2p}(t) \doteq \frac{1}{2i}\,\frac{e^{i(\omega_{21}-\Omega)t}-1}{(\omega_{21}-\Omega)}\cdot H_{fi} + \text{Regular term}$$

$$|C_{2p}(t)|^2 = \frac{\sin^2\left(\frac{\omega_{21}-\Omega}{2}\right)t}{(\omega_{21}-\Omega)^2}\,H_{fi}^2, \quad \text{Amplitude} = \frac{H_{fi}^2}{(\omega_{21}-\Omega)^2}$$

$$\approx \left(\frac{e E_z a_{Bohr}}{\omega_{21}-\Omega}\right)^2$$

Estimate:

$$|\omega_{21}-\Omega| \le (90\%)^{-1/2}\cdot e\, a_{Bohr} E_z$$

$$\approx 10^{+8}\,\text{Hz} < 1\,\text{GHz}$$

Recall $\omega_{21} \approx 10\,\text{eV} = 10^{16}\,\text{Hz}$ or $10^7\,\text{GHz}$

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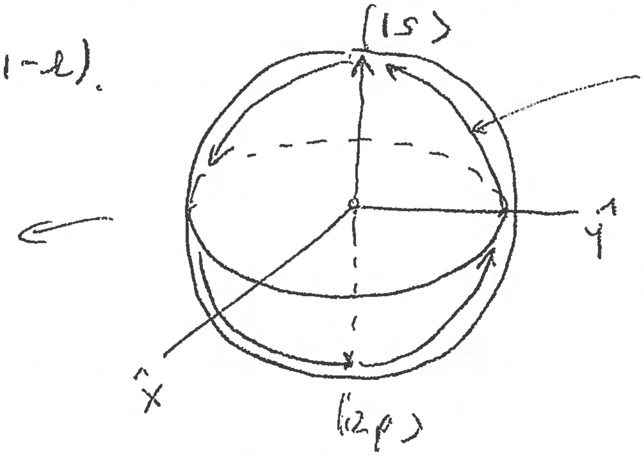
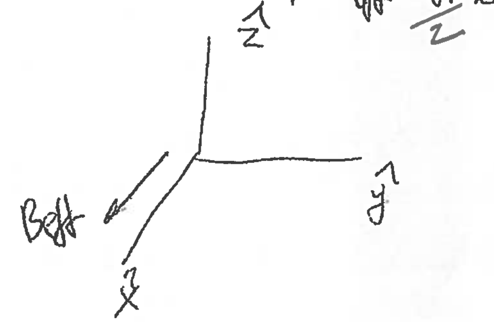
strictly speaking, one should consider the Rabi oscillations as $H_{fi} \gg |\omega_{21}-\Omega|$.

At $\Omega = \omega_{21}$, $\vec{B}_{eff} = \frac{H_{fi}}{2}\vec{e}_z$

Bloch Sphere





Prob. 2.

Emission Rate $A = \frac{\omega_0^3 |\wp|^2}{3\pi \varepsilon_0 \hbar c^3}$

$\hbar\omega_0 \sim 10\,eV$, $|\wp| \doteq e\, a_{Bohr}$

Putting everything together Emission Rate $= 10^9/s$

Life time $\sim 10^{-9} s$