Phys525:
Quantum Condensed Matter Physics:
emergent symmetry and phenomena

Topological States, Topological ordered states and SPT


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What do we mean by saying topological matter?

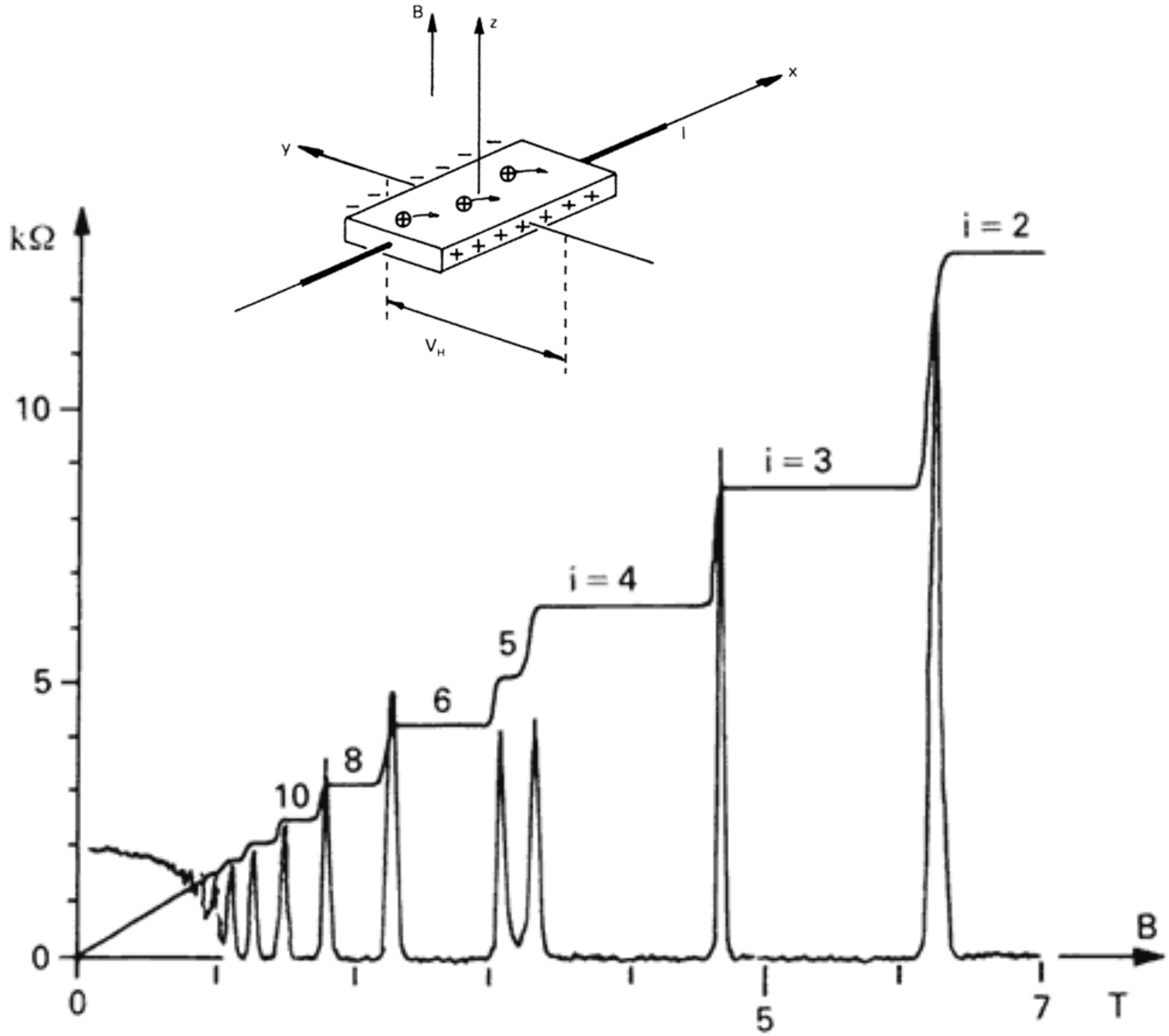
- Historically…

- before 80s in CMP: Topological defects (such as vortices, hedgehogs, textures), see Volovik, Mineev, 77; Mermin, 77; Anderson, Toulouse, 77)

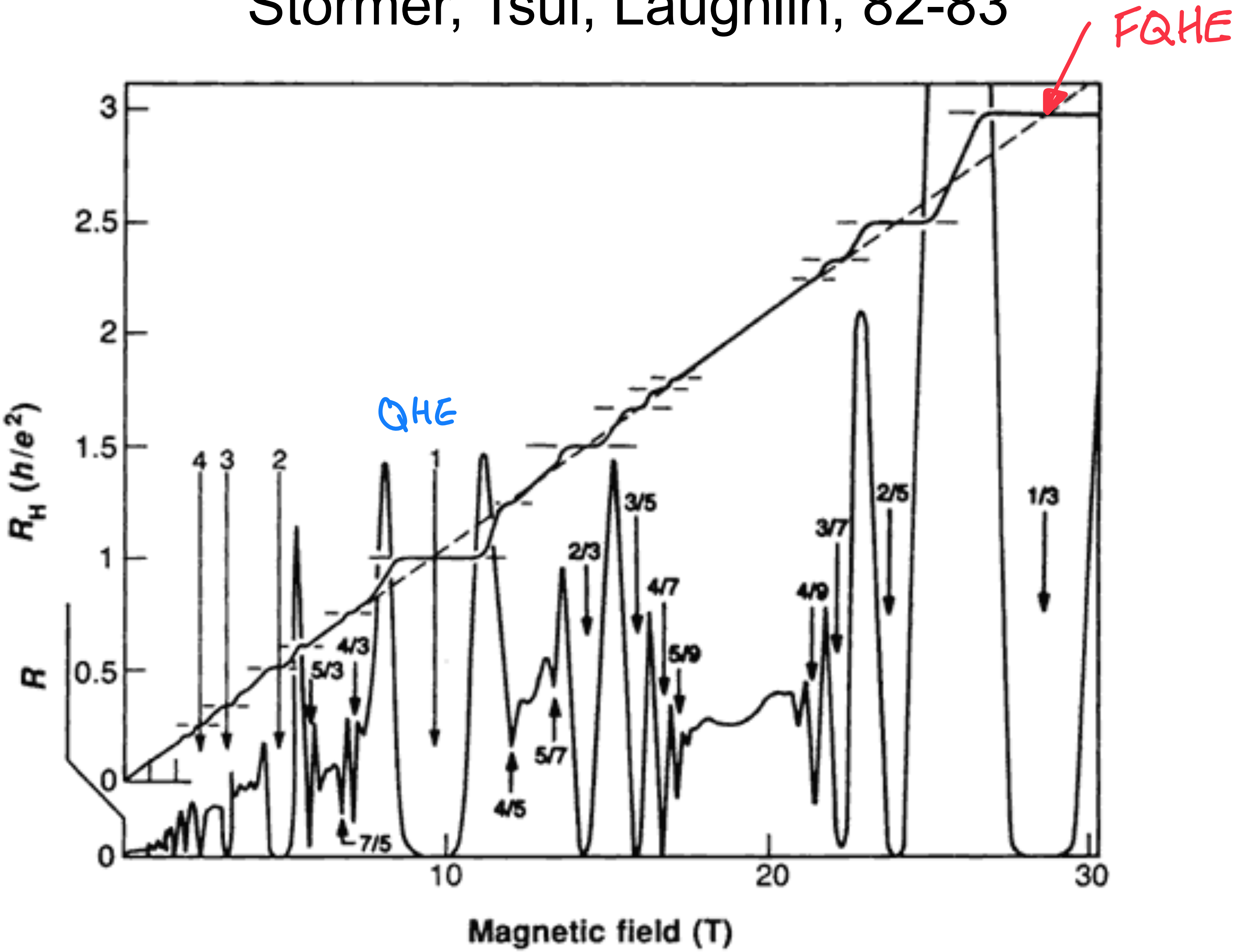
- KT or BKT Topological phase transitions, 72; transition driven by deconfinement dynamics.

- 80s and 90s were very exciting years
- QHE, quantized Hall conductance/first class Chern-number characterization of electronic states by TKNN.
- FQHE, Abelian anyons by Laughlin et al.
- Spin liquids as topologically ordered states.

QHE (Krause Von Klitzing, 80)



Störmer, Tsui, Laughlin, 82-83



- Two important concepts
- Boundary dynamics are important; bulk-edge correspondence emerged during that time (mainly in the context of FQHE).

- “Topological degeneracy”—-that ground states (with TS) can be degenerate. And if not (and if no TS symmetry breaking), non-topological.

- Affleck-Lieb-Mattis theorem on s=1/2 non-topological gapped spin states: breaking Translation symmetry.

Topological insulators and superconductors (2005~) (back to 10-fold way later)

- topological insulators without symmetry or with Z_1 symmetry—QHE

- Topological Insulator with time reversal symmetry —Z_2 topological Insulator

- Topological superfluids without symmetry or with Z_1 symmetry—-p+ip

- topological Superconductors with time reversal symmetry—-(p+ip) X (p-ip)

Topological ordered states vs Symmetry protected topological states (~2010)

- Topologically ordered states with long range entanglements, or a topological entropy;
- Symmetry protected topological states with short range entanglement entropy.

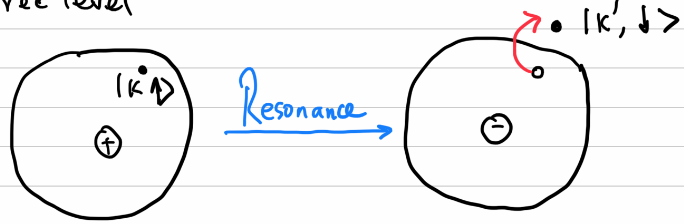
Supplementary Kondo effect

Kondo effect as Many-body Resonance

Approach 1 (Project into $|-,\uparrow\rangle, |+,\downarrow\rangle$) Spin projection

Direct interaction
tree level

One-loop



$$H_{kondo} = \frac{1}{2} J \underbrace{\left(S^+ s^- + S^- s^+\right)}_{\text{Fluctuations}} + J\, S_z s_z \qquad (J > 0)$$

$$\underbrace{\begin{cases} +\frac{J}{4} \quad |+,\uparrow\rangle \quad |-,\downarrow\rangle \\ -\frac{J}{4} \quad |-,\uparrow\rangle \text{ or } |+,\downarrow\rangle \end{cases}}_{\text{tree level}}$$

$= G^{(4)}$ (1PI)

$$J = -\frac{J(1)}{4} + \frac{J^2}{4} \underbrace{i \int G_I(\omega)\, G_e(\Omega - \omega, k'')\, \frac{d\omega}{2\pi} \frac{d^d k''}{(2\pi)^d}}_{I}$$

$\Omega = \varepsilon_{k\uparrow}$

$$I(\Omega \to 0) \simeq \frac{J^2}{4} \int G_e(0, k'')\, \frac{d^d k''}{(2\pi)^d}$$

$$I(\Omega \to 0) \simeq \frac{J^2}{4} \int G_e(0, k'') \frac{d^d k''}{(2\pi)^d}$$

$$= -\frac{J^2}{4} \int^{\Lambda} \frac{1}{\varepsilon_{k''}} D(\varepsilon_F)\, d\varepsilon_{k''} \qquad \varepsilon_{k''} = \frac{k^2}{2} - \mu$$

$$\simeq (-)\frac{J^2}{4} \ln \frac{\Lambda}{\epsilon} \to \infty \qquad < 0$$

Back on envelop RGE $\qquad \tilde{J}(\Lambda) = J(\Lambda) D(\varepsilon_F)$

$$\Lambda \frac{d}{d\Lambda} G^{(4)} = 0 \Rightarrow \frac{\partial \tilde{J}(\Lambda)}{\partial t} = -\tilde{J}^2$$

Λ ~ energy cut-off

$$\Lambda \frac{\partial J(\Lambda)}{\partial \Lambda} = -J^2 D(\varepsilon_F)$$