Phys529B: Topics of Quantum Theory

Lecture 14: identifying SIFP via Callan Symanzik RGE

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- The basic idea of renormalizability: 1) physical measureables shall be independent of the UV scale at which you formulate a theory; I.e. “scale independent”.   2) there can be infinite numbers of field theories of one single physical reality; they are all equivalent. 3) different theories is connected by Renormalization group equations.

Supplementary stuff on RGE

$$H = H(m, \lambda, \ldots ; \Lambda) = H(\tilde{m}, \tilde{\lambda}, \cdots, Z(\Lambda) ; \Lambda)$$

for any given Λ, there shall be a $\tilde{m}(\Lambda)$, $\tilde{\lambda}(\Lambda)$ ...

so that H leads to the same physics.

$$\frac{d\tilde{m}}{dt} = \beta_m(\tilde{m}, \tilde{\lambda}, \ldots) \qquad \frac{dZ(\Lambda)}{dt} = \gamma(\tilde{m}, \tilde{\lambda}, Z)$$

$$\frac{d\tilde{\lambda}}{dt} = \beta_{\tilde{\lambda}}(\tilde{m}, \tilde{\lambda}, \ldots) \qquad t = \ln\frac{\Lambda}{\Lambda_{uv}}$$

Renormalization Group Equations

Running scale

Λ_{UV}	$\phi_0(x)$	$H(g_{01}, g_{02}, \dots; m, \Lambda_{UV})$
Λ	$\phi(x)$	$H(g_1(\Lambda), g_2(\Lambda), \dots; m(\Lambda), \Lambda; Z(\Lambda))$
	$\phi(x) = Z^{-1/2}\phi_R(x)$	
	or $= Z^{1/2}\phi_0(x)$	
Λ_{IR}	$\phi_R(x)$	$H(g_{1R}, g_{2R}, \dots; m_R)$

- Callan-Symanzik approach (also Coleman-Weinberg applications) : Utilizing Green’s functions to obtain RGEs and understand scale Transformations

- An interesting application to cold gases can be found in Yang, Jiang and Zhou, PRL 124 (22), 225701, 2020, Tricritical physics in 2D Superfluids; Jiang and Zhou, ArXiv:2301.12657 at https://arxiv.org/pdf/2301.12657.pdf, Interacting 3D Fermions in Planckian limit: Entropy and dynamics.

Approach 1: $\quad \phi(x) = Z^{1/2}(\Lambda)\, \phi_R(x)$ $\quad$ IPR

$$\langle 0 | T \phi(x_1) \phi(x_2) \cdots \phi(x_n) | 0 \rangle = G^{(n)}(x_1, \ldots x_n)$$

$$G^{(n)}(x_1, \ldots x_n) = Z^{n/2}(\Lambda) \underbrace{\langle 0 | T \phi_R(x_1) \cdots \phi_R(x_n) | 0 \rangle_c}_{\text{independent of "}\Lambda\text{"}}$$

Computed with $H(g_1(\Lambda), g_2(\Lambda), \cdots g_n(\Lambda); m(\Lambda), \Lambda)$

$$G^{(n)} = G^{(n)}(\ldots ; \tilde{g}_1, \tilde{g}_2, \ldots \tilde{g}_n(\Lambda); \tilde{m}(\Lambda), t)$$

$$t = \ln \Lambda$$

$$\left\{ \frac{\partial}{\partial t} + \beta(g) \frac{\partial}{\partial g} \right\} G^{(n)} = \left\{ \frac{n}{2} \underbrace{\Lambda \frac{\delta Z}{\delta \Lambda} \frac{1}{Z}}_{\gamma(\tilde{g})} \right\} G^{(n)}$$

$$\left\{ \frac{\partial}{\partial t} + \beta(g) \frac{\partial}{\partial g} - \frac{n}{2} \gamma(g) \right\} G^{(n)} = 0$$

Note: $Z(t=0, \tilde{g}) = 1$

Approach II : $\phi(x) = Z^{-1/2}(\Lambda)\,\phi_0(x)$

$$\langle 0|\,T\,\phi(x_1)\phi(x_2)\cdots\phi(x_n)\,|0\rangle = G^{(n)}(x_1,\dots x_n)$$

$$G^{(n)}(x_1,\dots x_n) = Z^{-n/2}(\Lambda)\,\underbrace{\langle 0|\,T\,\phi_0(x_1)\cdots\phi_0(x_n)|0\rangle_c}_{\text{independent of ``}\Lambda\text{''.}}$$

Computed with $H(g_1(\Lambda), g_2(\Lambda), \cdots g_n(\Lambda); m(\Lambda), \Lambda)$

$$G^{(n)} = G^{(n)}(\cdots;\ \tilde{g}_1, \tilde{g}_2, \dots \tilde{g}_n(\Lambda); \tilde{m}(\Lambda), t)$$

$$t = \ln\Lambda$$

$$\left\{ \frac{\partial}{\partial t} + \beta(g) \frac{\partial}{\partial g} \right\} G^{(n)} = \left\{ \underbrace{\frac{n}{2} \Lambda \frac{\delta Z}{\delta \Lambda} \frac{1}{Z}}_{\gamma(\tilde{g})} \right\} G^{(n)}$$

$$\left\{ \frac{\partial}{\partial t} + \beta(g) \frac{\partial}{\partial g} + \frac{n}{2} \gamma(\tilde{g}) \right\} G^{(n)} = 0$$

Note : $Z\left(t = \ln \frac{\Lambda_{UV}}{\Lambda_{IR}}, \tilde{g}\right) = \mathbb{1}$

Relations between Approach I and Approach II

App I: $\phi(x) = Z_{I}^{1/2}\,\phi_R(x)$; App II: $\phi(x) = Z_{II}^{-1/2}\,\phi_0(x)$

$$Z_{I}^{1/2}\, Z_0^{-1} = Z_{II}^{-1/2} \quad \text{or}$$

$$\boxed{\frac{1}{Z_{I}}\frac{\partial}{\partial t} Z_{I} = -\frac{1}{Z_{II}}\frac{\partial}{\partial t} Z_{II}}$$

$$\left\{ \frac{\partial}{\partial t} + \beta(g) \frac{\partial}{\partial g} \right\} G^{(n)} = \left\{ \underbrace{\frac{n}{2} \Lambda \frac{\delta Z}{\delta \Lambda} \frac{1}{Z}}_{\gamma(\tilde{g})} \right\} G^{(n)}$$

$$\left\{ \frac{\partial}{\partial t} + \beta(g) \frac{\partial}{\partial g} - \gamma(\tilde{g}) \right\} G^{(n)} = 0$$

Note: $Z(t=0, \tilde{g}) = 1$, $\tilde{g} = g \Lambda^{d-3}$

How to use them?

two-point $\quad n=2$

I: $\left\{\frac{\partial}{\partial t} + \beta(g)\frac{\partial}{\partial g} - \gamma(\tilde{g})\right\} G(p) = 0\,, \quad G^{(2)}(p) \sim \frac{Z}{p^2}$

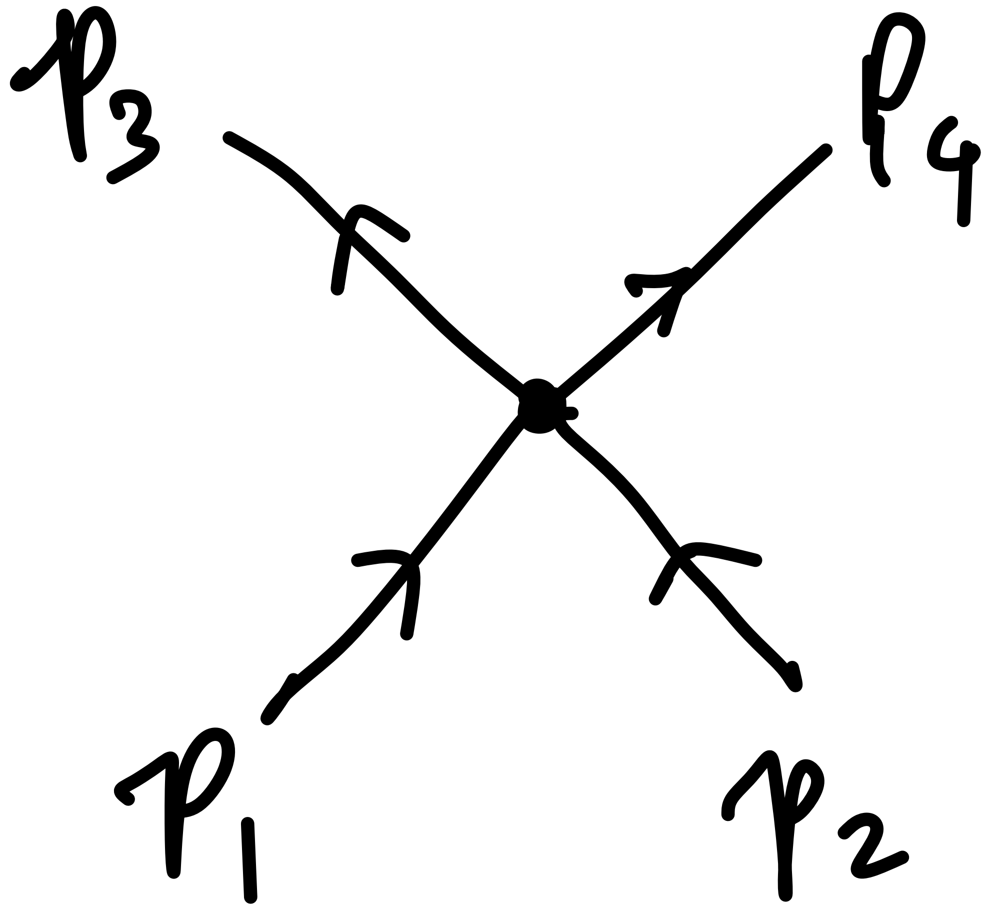
$$Z(\Lambda = \Lambda_{2R}, \tilde{g}) = \mathbb{1} \longrightarrow \frac{\partial}{\partial \tilde{g}} G^{(2)}(p) = 0$$

$$\gamma(\tilde{g}) = G(p)^{-1} \frac{\partial}{\partial t} G(p) = \frac{1}{Z(\Lambda)} \frac{\partial}{\partial t} Z(\Lambda)$$

RGE $\qquad$ Green-function

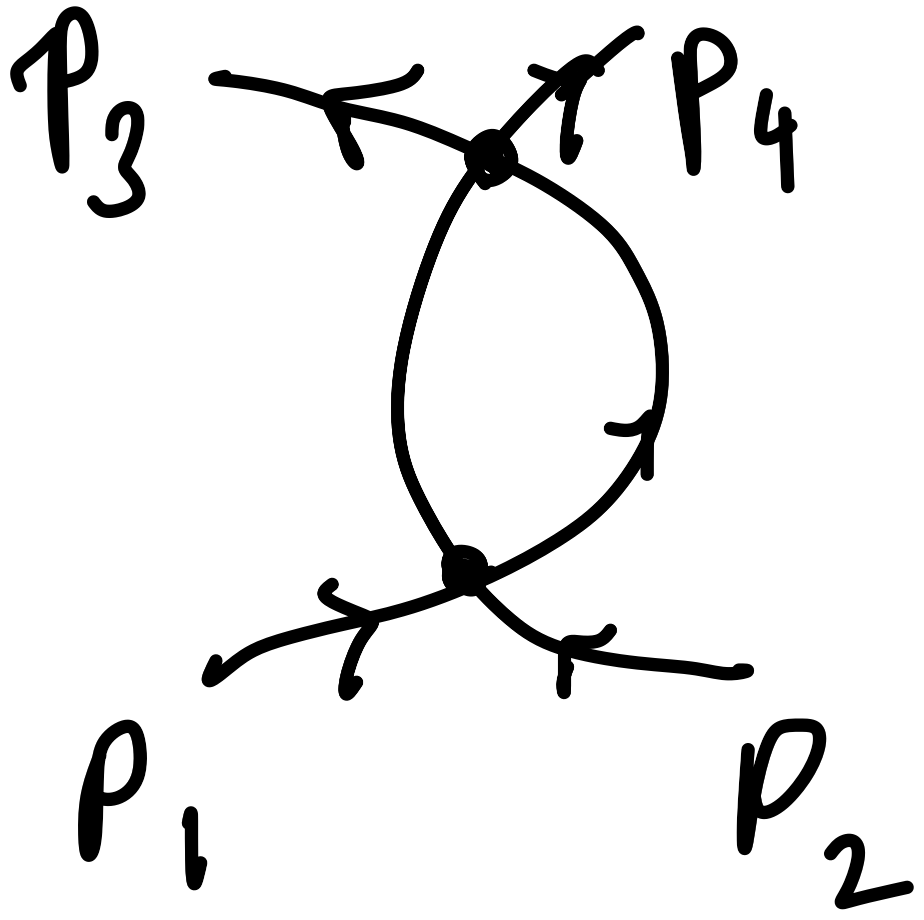
$$\mathrm{II}: \left\{ \frac{\partial}{\partial t} + \beta(g)\frac{\partial}{\partial g} - 2\gamma(\tilde{g}) \right\} G(p_1, p_2, p_3, p_4) \doteq 0$$

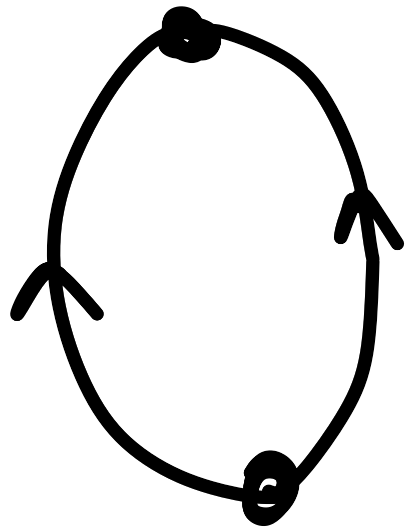
4-point 1PI



$$\frac{d\tilde{g}}{dt} = \beta(\tilde{g}) = 2\gamma(\tilde{g})\,\tilde{g} + \tilde{g}(d-3) - \frac{\partial}{\partial t}\left[X \right]$$

Λ^{d-3}



X =  + ...

1PI

$$X = -g^2 \int_{\Lambda_{IR}}^{\Lambda} \frac{1}{(\omega^2 + q^2)^2} \frac{d\omega}{2\pi} \frac{d^{(d)}\underline{q}}{(2\pi)^d}$$

$$\frac{\partial X}{\partial t} \simeq -\frac{\Omega_{d+1}}{(2\pi)^{d+1}} g^2 \Lambda^{d-3}$$

$$c_d \tilde{g}^* = +(3-d)$$

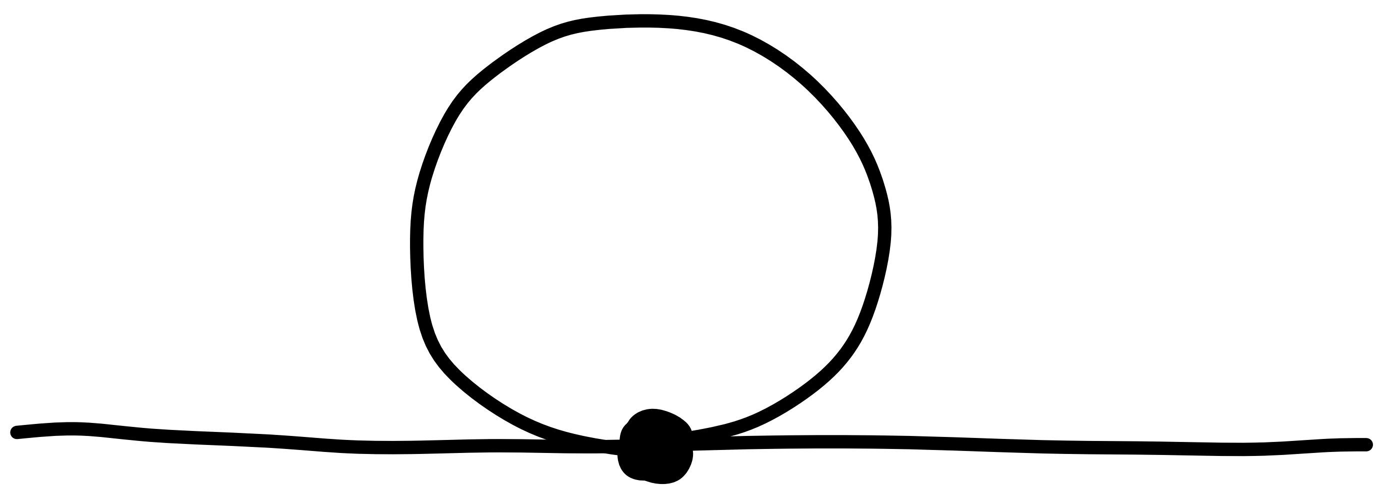
for $3 > d$

$$-\Lambda^{d-3} \frac{\partial X}{\partial t} = +\frac{\Omega_{d+1}}{(2\pi)^{d+1}} \tilde{g}^2$$

$$\beta(\tilde{g}) = (d-3)\,\tilde{g} + c_d\, \tilde{g}^2, \qquad c_d = \frac{\Omega_{d+1}}{(2\pi)^{d+1}}$$

$3 > d$

$$\frac{d\tilde{m}^2}{dt} = -2\tilde{m}^2 - b_d\,\tilde{g}\,, \qquad \tilde{m}^{2*} = -\frac{(3-d)}{2}$$



$$\delta m(\Lambda) = g\int^{\Lambda} \frac{1}{\omega^2+q^2}\,\frac{d\omega}{2\pi}\,\frac{d^d\vec{q}}{(2\pi)^d}$$

$$b_d = \frac{\Omega_{d+1}}{(2\pi)^{d+1}} = C_d$$