Phys525:
Quantum Condensed Matter Physics:
Emergent symmetries in CMP

Episode 10:

Quantum coarse graining and (d+1) dimension Theory II:

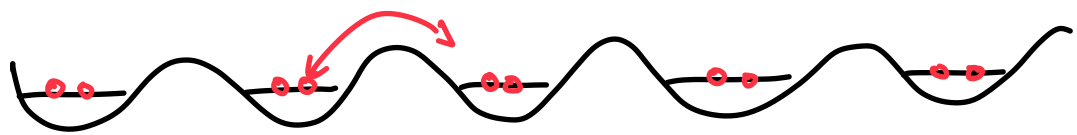
From Quantum Particles to Quantum Fields via the method of embedding

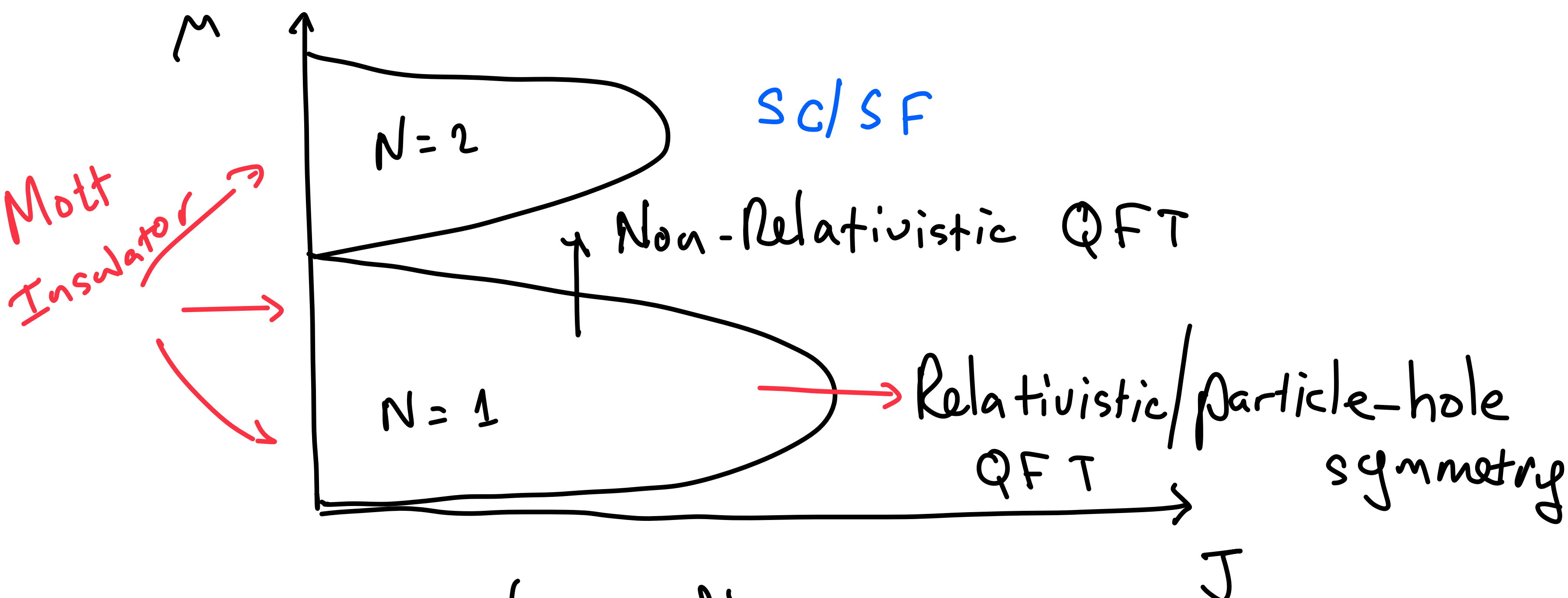
Quantum Model II : $[b_i, b_j] = 0$, $[b_i, b_j^\dagger] = \delta_{ij}$

$$\mathcal{H}_{BH} = \sum_i \frac{\hat{N}_i(\hat{N}_i - 1)}{2c} - \mu \hat{N}_i - J \sum_{\langle ij \rangle} b_i^\dagger b_j + h.c.$$

$(\hat{N}_i = b_i^\dagger b_i)$

Bose-Hubbard Model



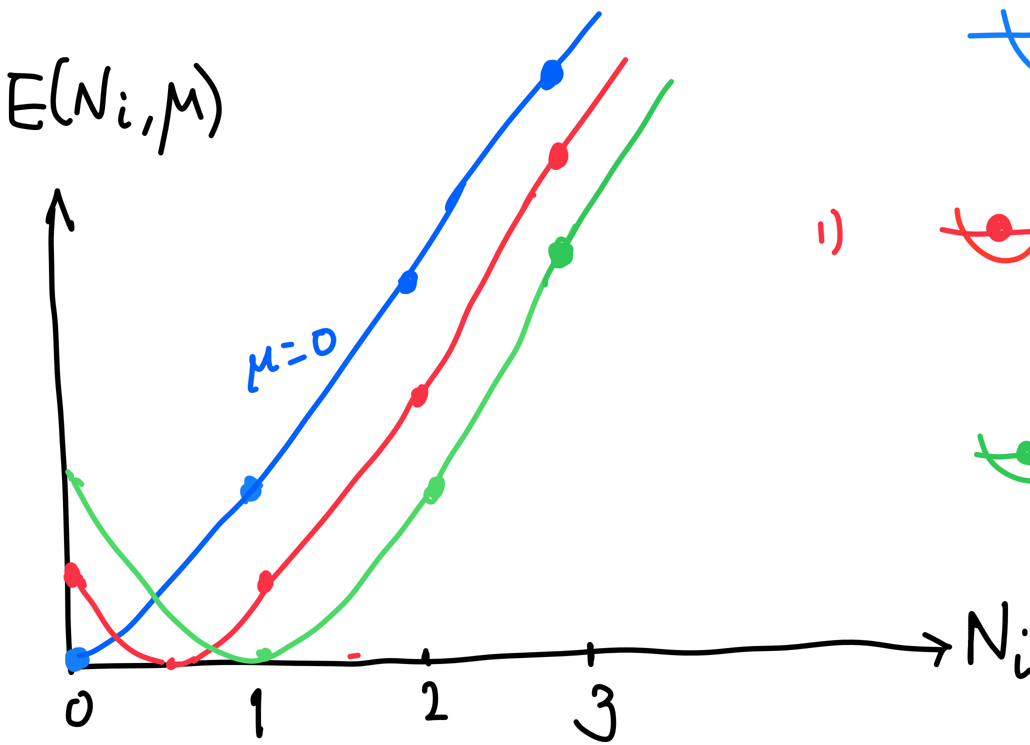


T=0 phase diagram

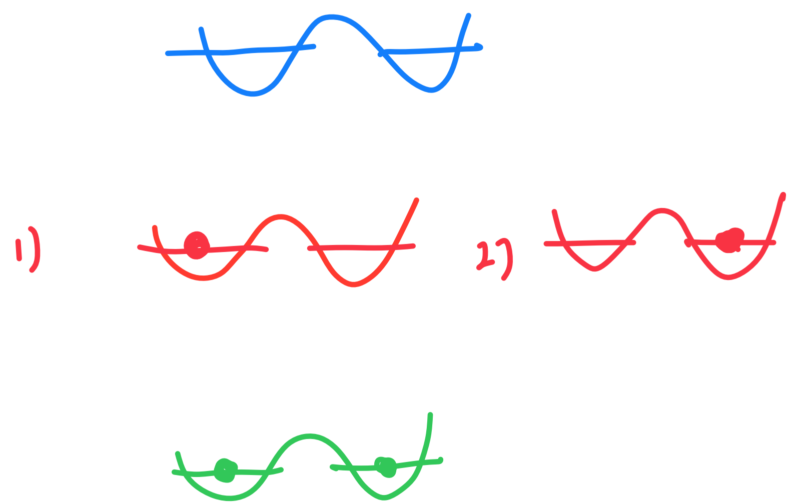
Quantum Model II:

$$\mathcal{H}_{BH} = \sum_i \frac{\hat{N}_i^2}{2c} - \mu \hat{N}_i + \overset{J}{\cancel{0}}$$

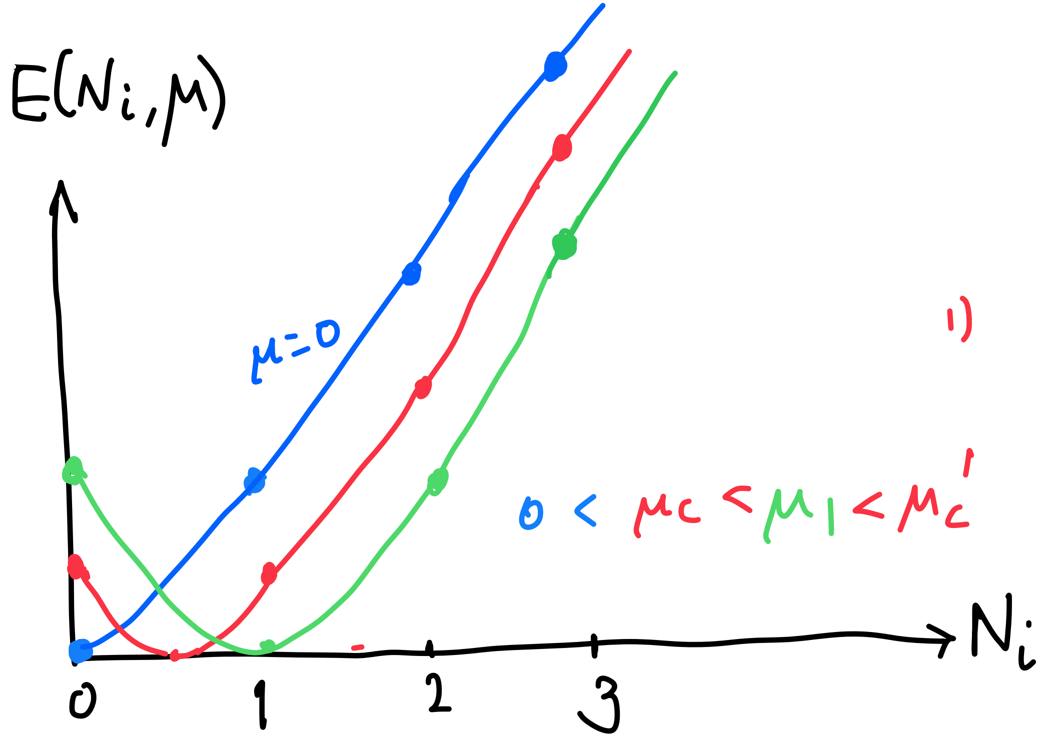
Bose-Hubbard Model

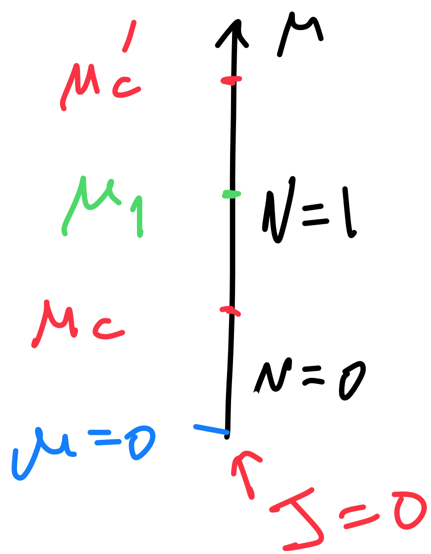


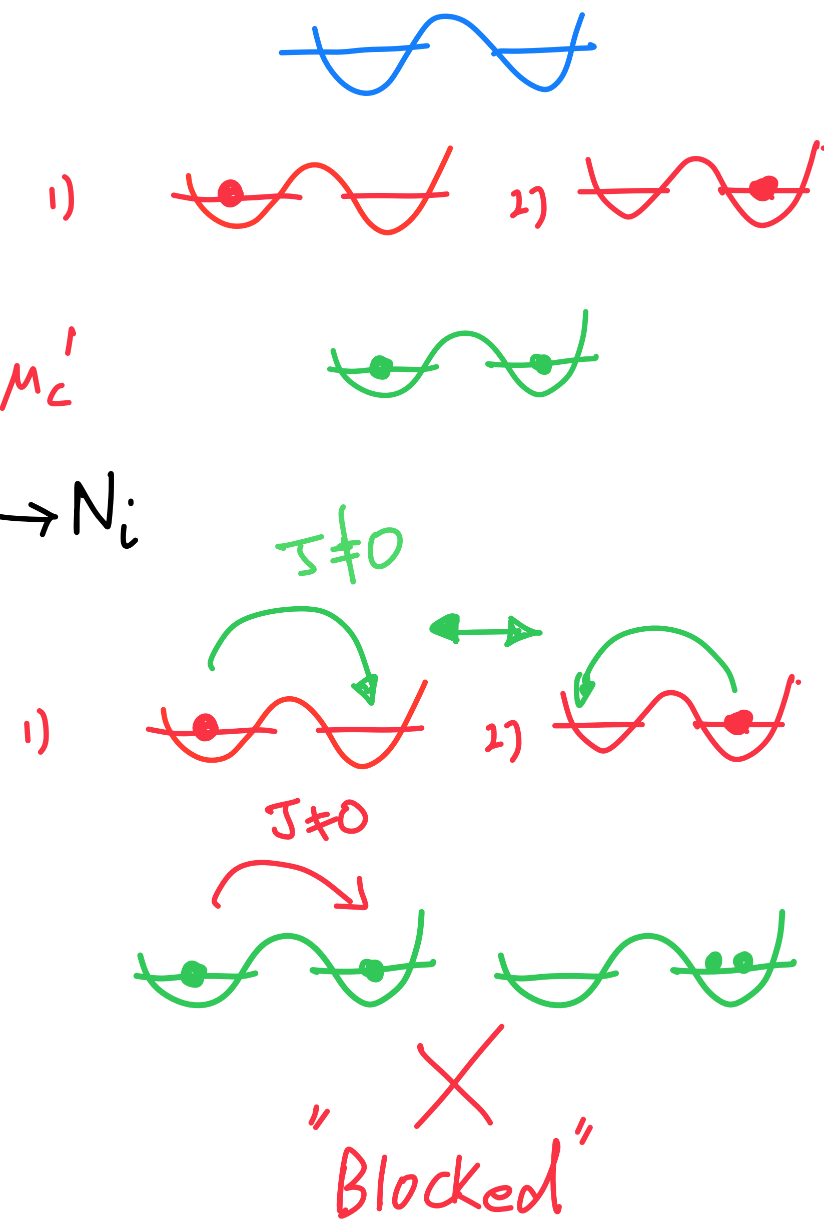
Two site physics

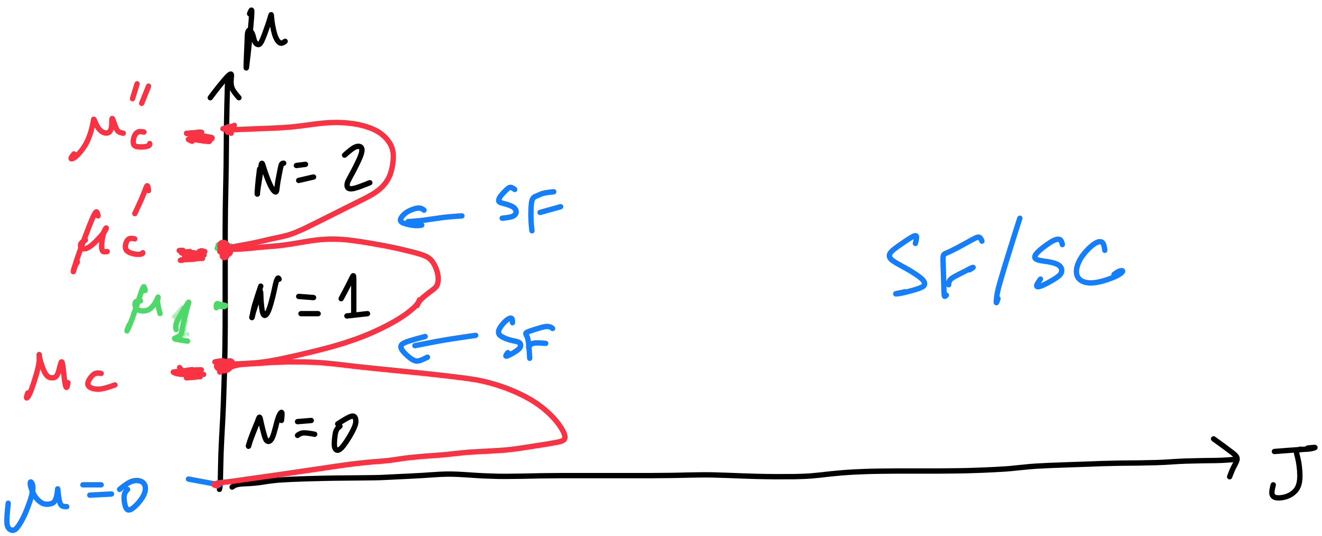


Two site physics



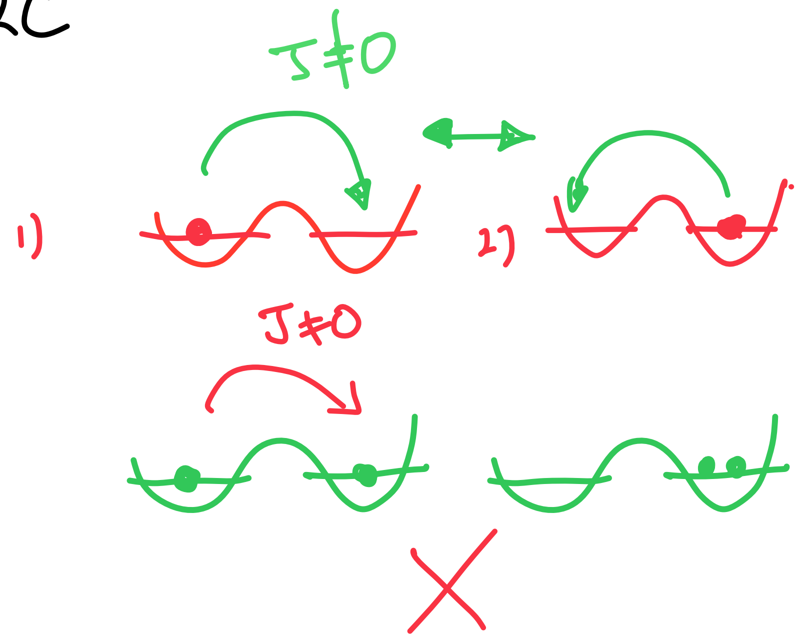






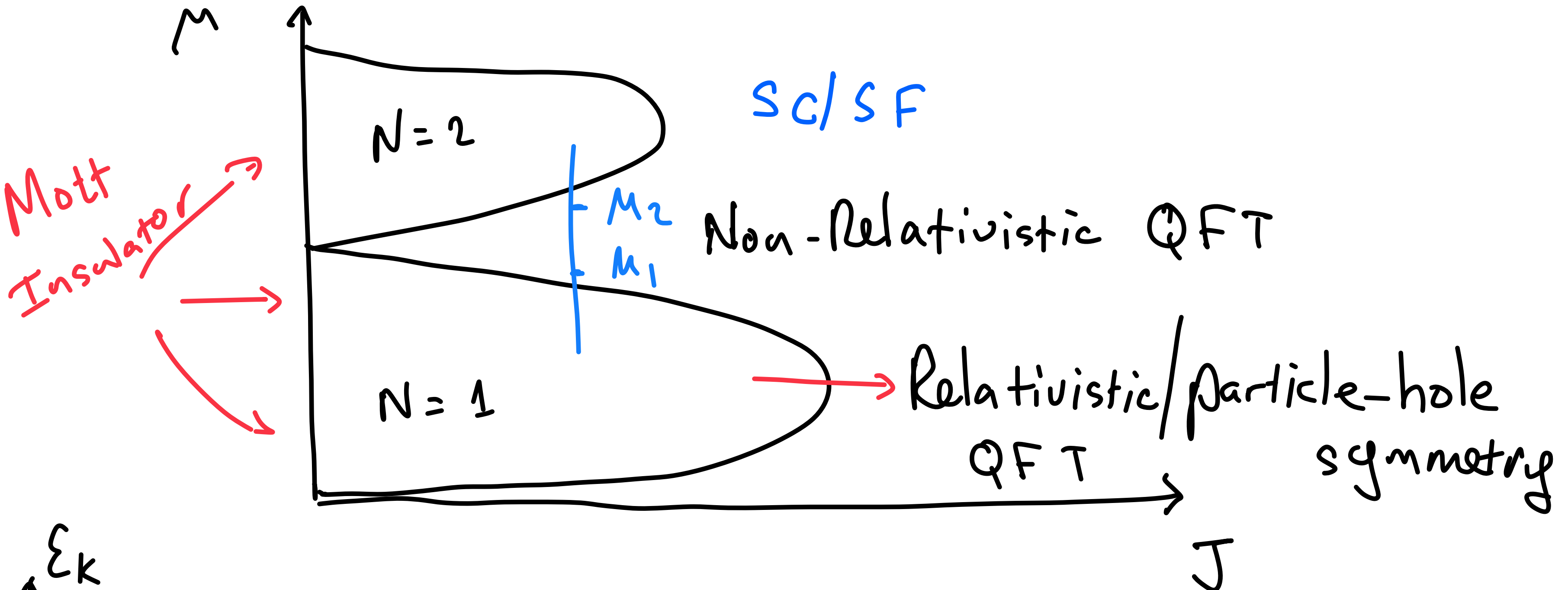
$$\frac{1}{2C} = U \gg J \to 0$$

$$0 \leftarrow \frac{1}{2C} = U \ll J$$

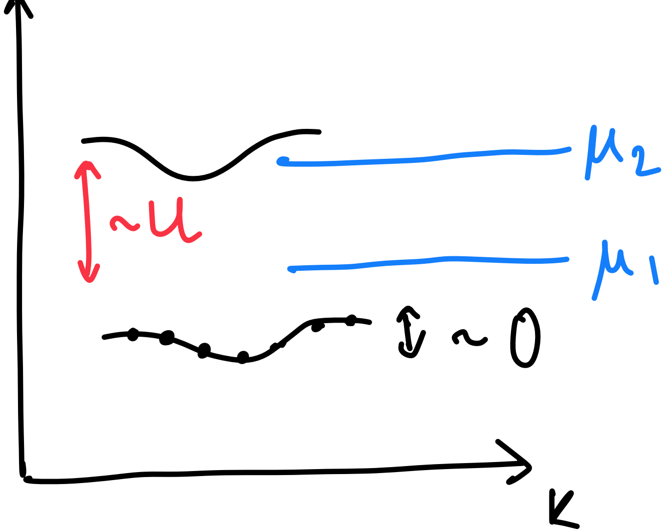


$$\mathcal{H} = -J \sum_{\langle ij \rangle} b_i^{+} b_j + h.c.$$

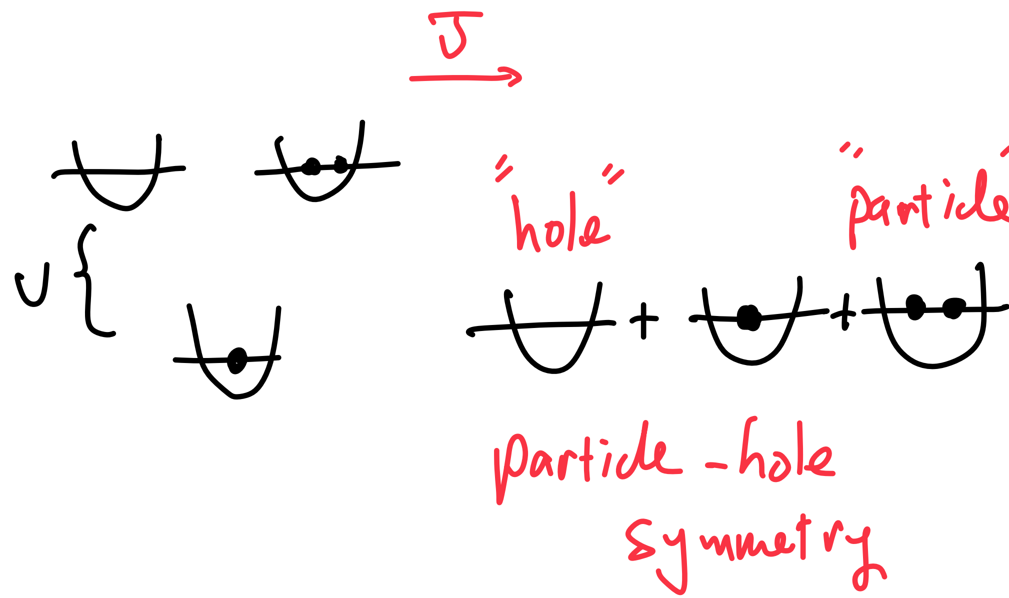
"free boson Model"



μ
J
$N=2$
$N=1$
SC/SF
μ_2
μ_1
Mott Insulator
Non-Relativistic QFT
Relativistic QFT
/ particle-hole symmetry

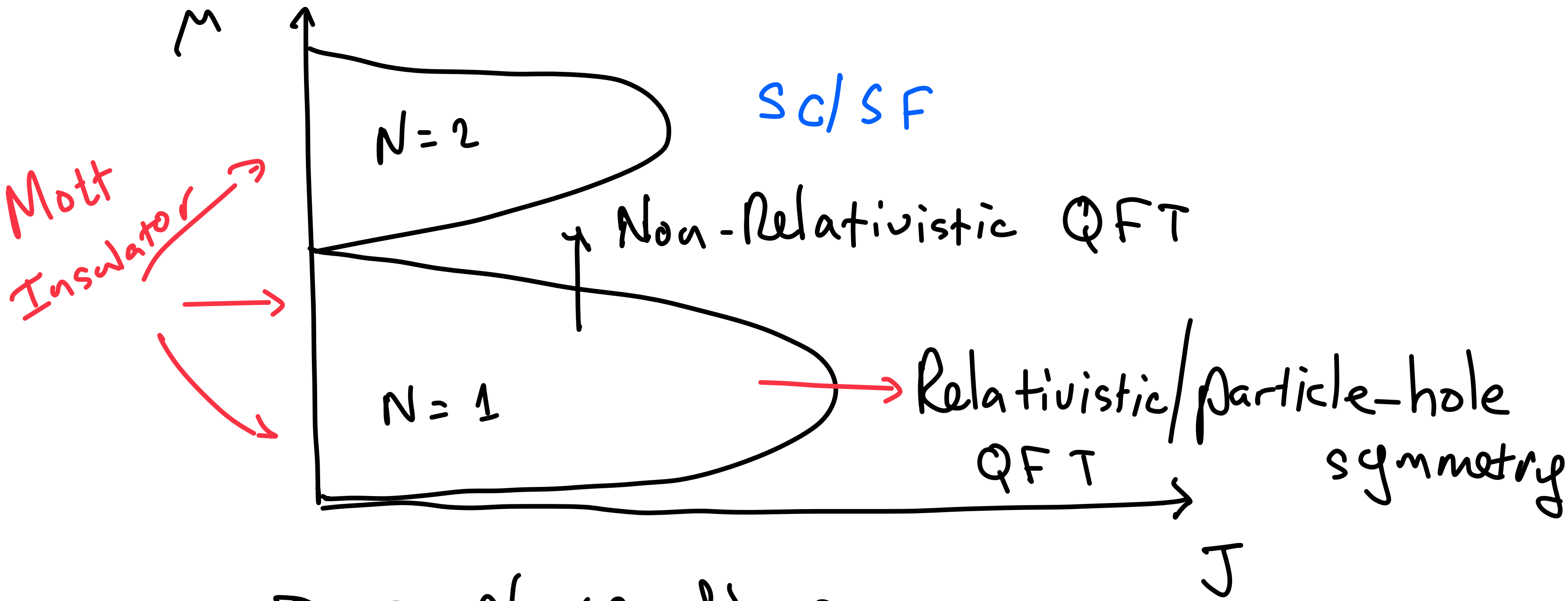


ε_k
k
$\sim U$
μ_2
μ_1
~ 0



J
U
"hole"
"particle"
+
+
particle-hole symmetry

Quantum Model II :



T = 0 phase diagram

mappings

- From the Bose-Hubbard model (BHM) (D=d) to an XY spin model (D=d+1).

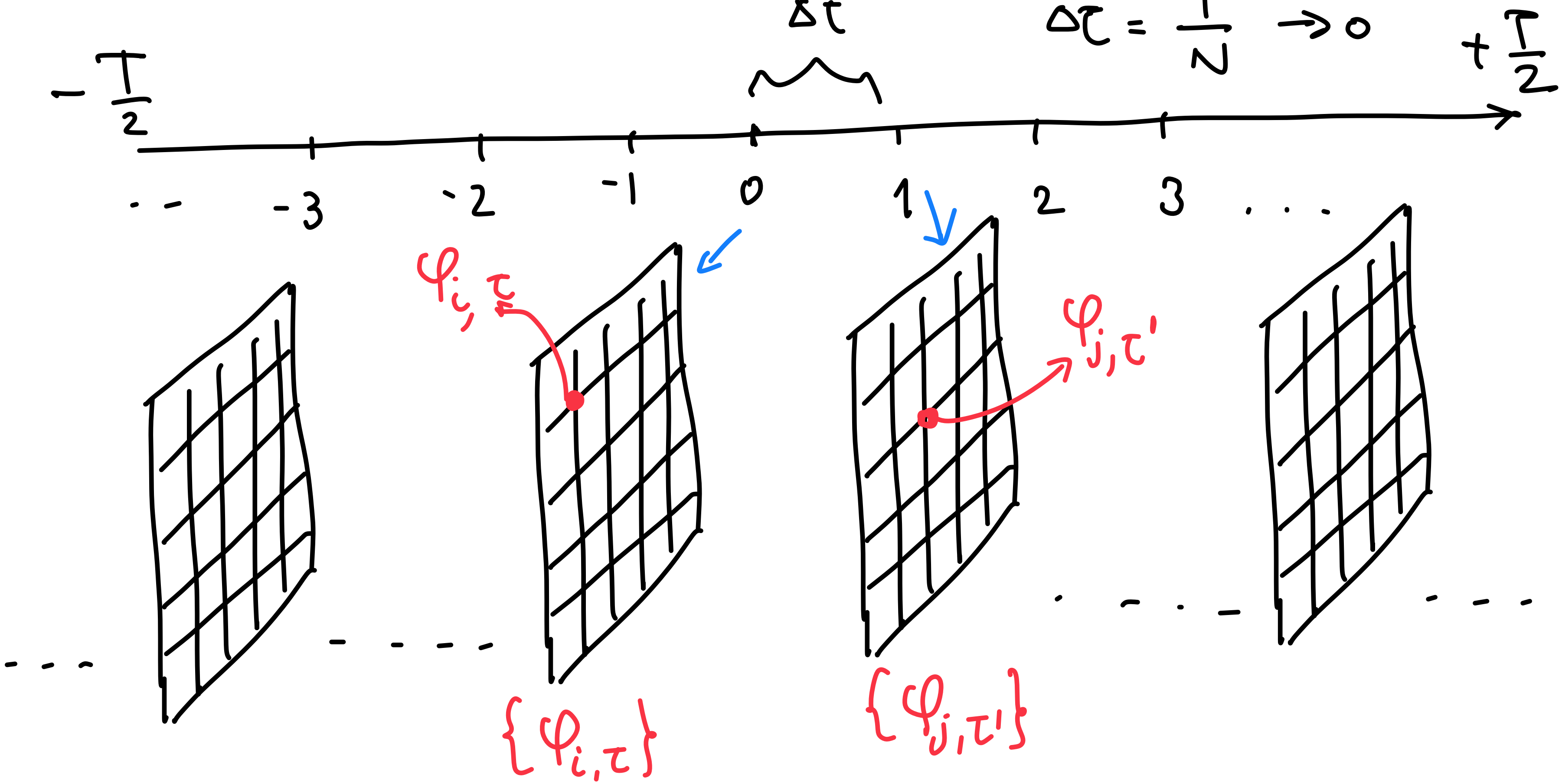
- From the XY spin model to QFT of a complex bosonic field via ***a method of embedding,*** *a very powerful method.*

$$H_{BHM}(D = d) \to \beta H_{XY}(D = d + 1) \to H_{qft}(\psi \in \mathscr{C})(D = d + 1)$$

Imaginary time evolution

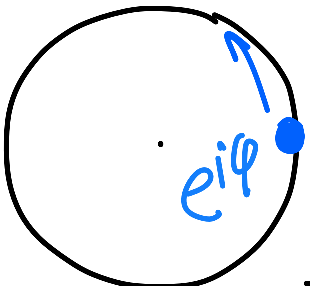
$$Z = \langle 0 | e^{-HT} | 0 \rangle$$

$$\Delta\tau = \frac{T}{N} \to 0$$



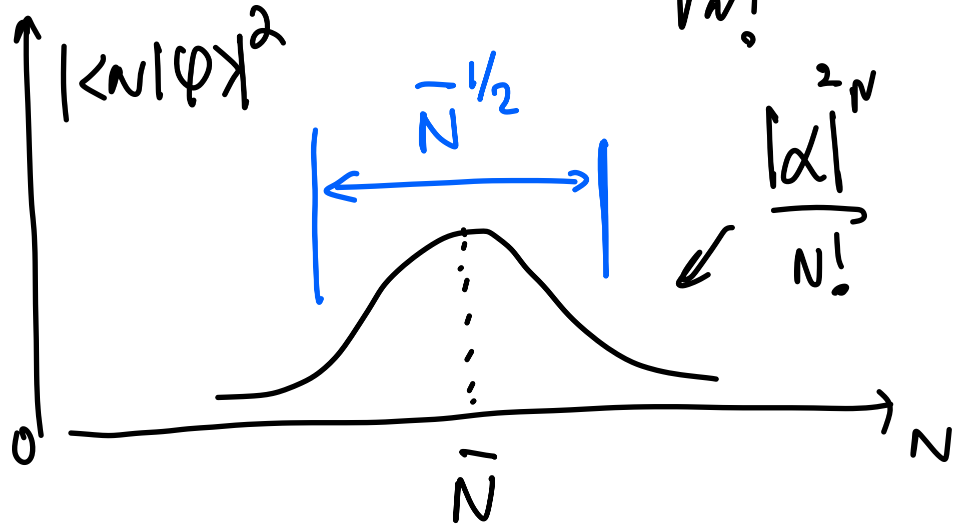
$$Z = \sum_{\{\varphi_{i,\tau}\}} \cdots \sum_{\{\varphi_{i,0}\}} \sum_{\{\varphi_{j,1}\}} \cdots \sum_{\{\varphi_{i,\tau}\}} \cdots \langle \{\varphi_{j,1}\} | e^{-H\Delta\tau} | \{\varphi_{i,0}\} \rangle \cdots$$

"Complete Set for Quantum Coarse graining with Larg $\bar{N}$"



$|\varphi\rangle \sim e^{N_0^{1/2} e^{i\varphi} b^{+}}|vac\rangle$, $\quad |N\rangle \sim \frac{b^{+N}}{\sqrt{N!}}|vac\rangle$

$$|\varphi\rangle = e^{\alpha b^{+} - \frac{\alpha\alpha^*}{2}}|0\rangle$$



$$\frac{|\alpha|^{2N}}{N!}$$

a) $b|\varphi\rangle = N_0^{1/2} e^{i\varphi}|\varphi\rangle$

$$\langle\varphi|b^{+}b|\varphi\rangle = N_0 \gg 1$$

$$|N\rangle \sim \int_0^{2\pi} d\phi\, e^{-iN\phi}|\phi\rangle$$

b) $\hat{N}|N\rangle = N|N\rangle$

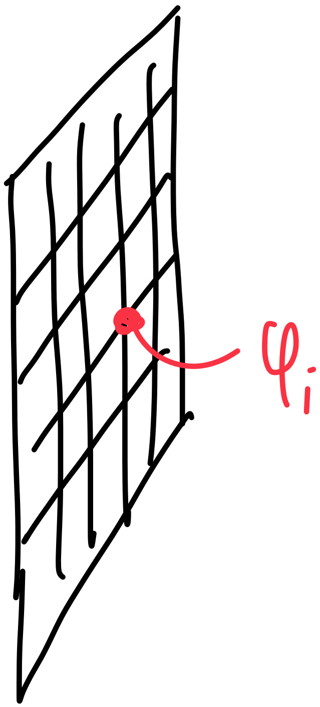
Fock states $\quad \hat{N}|\varphi\rangle = \frac{\partial}{i\partial\varphi}|\varphi\rangle$

"$\hat{N}$, φ are Conjugate."

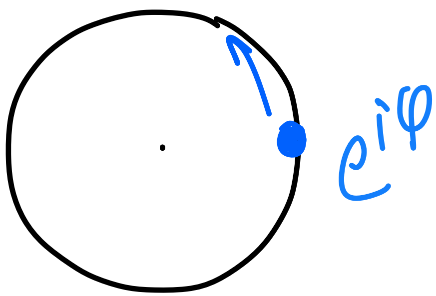
"Complete Set for Quantum Coarse graining with Large $\bar{N}$"

$$|\{\varphi_{i,\tau}\}\rangle = |\varphi_1,\tau\rangle \otimes |\varphi_2,\tau\rangle \otimes |\varphi_3,\tau\rangle \dots \otimes |\varphi_N,\tau\rangle$$

$$\begin{cases} \int D\phi_i \, |\{\varphi_{i,\tau}\}\rangle\langle\{\varphi_{i,\tau}\}| = \mathbb{1} \\ D\phi_i = \prod_{i=1}^{N} d\phi_i \end{cases}$$



"τth slice"



internal S^1
for site i

$$\begin{cases} \langle\varphi|\varphi'\rangle \simeq \delta_{\varphi,\varphi'} \\ b|\varphi\rangle = \bar{N}^{1/2} e^{i\varphi}|\varphi\rangle \\ \langle\varphi| b^\dagger b |\varphi\rangle = \bar{N} \gg 1 \end{cases}$$

$$|\varphi\rangle \sim e^{\bar{N}^{1/2} e^{i\varphi} b^\dagger} |Vac\rangle$$

$\cdots \quad -3 \quad -2 \quad -1 \quad 0 \quad 1 \quad 2 \quad 3 \quad \cdots$

$$b|\varphi\rangle \sim e^{i\varphi}|\varphi\rangle$$

$$Z = \sum_{\{\varphi_{i,\tau}\}} \cdots \sum_{\{\varphi_{i,0}\}} \sum_{\{\varphi_{i,1}\}} \cdots \sum_{\{\varphi_{i,\tau}\}} \cdots \underbrace{\langle \{\varphi_{i,1}\} | e^{-H\Delta\tau} | \{\varphi_{i,0}\} \rangle}_{T_{1,0}} \cdots$$

$$H = H_1 + H_2, \qquad H_1 = \sum_i \frac{\delta \hat{N}_i^2}{2C}, \qquad H_2 = -J \sum_{\langle ij \rangle} \underbrace{b_i^\dagger}_{e^{-i\varphi_{i,\tau}}} \underbrace{b_j}_{e^{i\varphi_{j,\tau}}} + h.c.$$

$$T_{1,0} = \underbrace{\langle \{\varphi_{i,1}\} | e^{-H_1\Delta\tau} | \{\varphi_{i,0}\} \rangle}_{e^{-\frac{C}{2}\left(\frac{\varphi_{i,1}-\varphi_{i,0}}{\Delta\tau}\right)^2 \Delta\tau} \cdots} \; e^{-H_2(\{\varphi_{i,0}\})\Delta\tau}$$

Hint:

$$\sum_{\delta N} \langle \varphi_1 | e^{-\frac{\delta \hat{N}^2}{2C}\Delta\tau} |\delta N\rangle\langle \delta N| \varphi_0 \rangle$$

approximate with $\int \delta N$

Imaginary time evolution

$$Z = \sum_{\{\varphi_{i,\tau}\}} \cdots \sum_{\{\varphi_{i,0}\}} \sum_{\{\varphi_{j,1}\}} \cdots \sum_{\{\varphi_{i,\tau}\}} \cdots \langle \{\varphi_{j,1}\} | e^{-H\Delta\tau} | \{\varphi_{i,0}\} \rangle \cdots$$

$$= \int D\varphi \, e^{-S(\{\varphi(\vec{x},\tau)\})}, \quad S = \int d\vec{x}^{\,(d)} \int d\tau \, \mathcal{L}(\{\varphi(\vec{x},\tau)\})$$

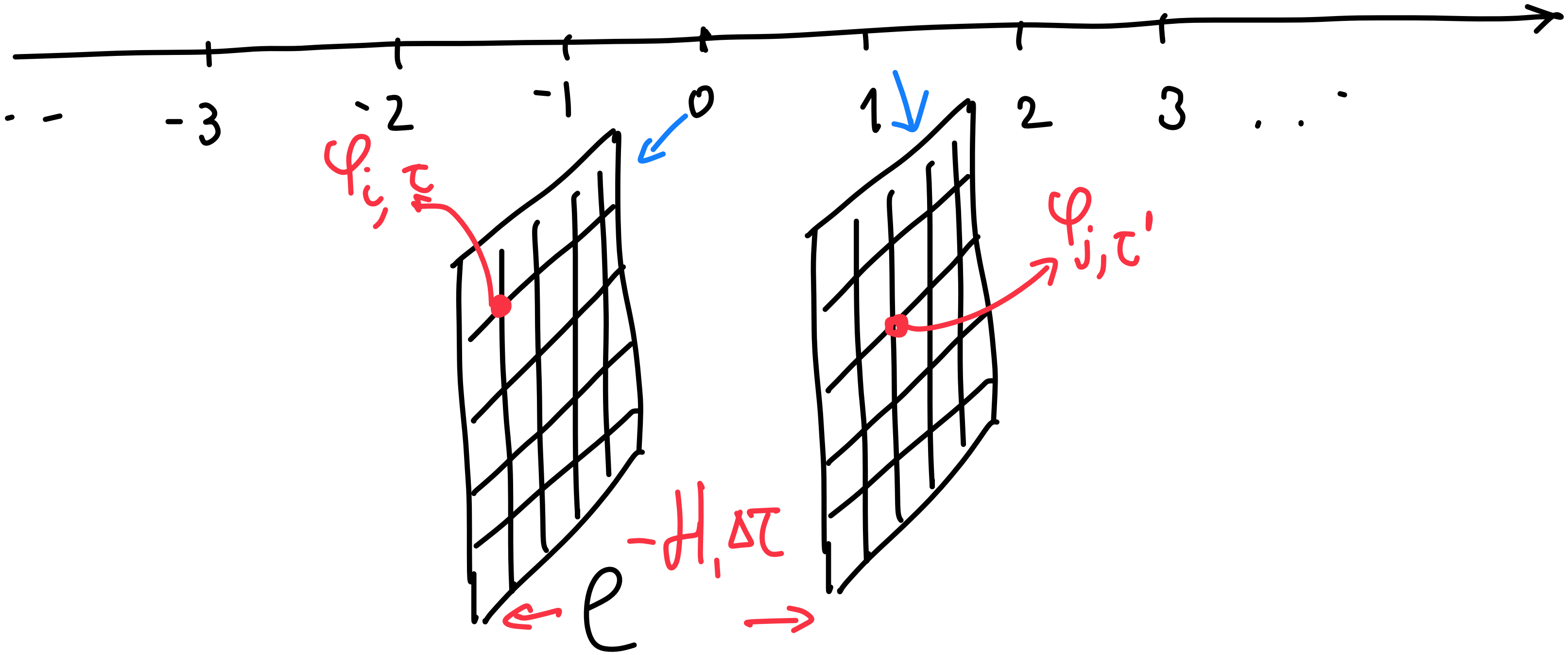
$\varphi(x,\tau)$ Action

$D\varphi$: functional integral

$$\mathcal{L} = \mathcal{L}_1 + \mathcal{L}_2$$

$$\mathcal{L}_1 = -\tilde{J}_\tau \cos(\varphi(\vec{x},\tau) - \varphi(\vec{x},\tau+\Delta\tau))$$

$$\mathcal{L}_2 = -\tilde{J}_{\vec{x}} \cos(\varphi(\vec{x}_i,\tau) - \varphi(\vec{x}_j,\tau))$$



$$S = \int d^{d_1}\vec{x} \int d\tau\, \mathcal{L}(\{\varphi(\vec{x},\tau)\})$$

$$\mathcal{L} = \mathcal{L}_1 + \mathcal{L}_2$$

$$\mathcal{L}_1 = -\tilde{J}_\tau \cos(\varphi(\vec{x},\tau) - \varphi(\vec{x},\tau+\Delta\tau))$$

$$\mathcal{L}_2 = -\tilde{J}_{\vec{x}} \cos(\varphi(\vec{x}_i,\tau) - \varphi(\vec{x}_j,\tau))$$

Eembedding in a QFT

$$Z_{XY}(\{\varphi_{\vec r,\tau}\}, \vec r \in R^d, \tau \in R) \Rightarrow Z(\{\phi(\vec r,\tau), (\vec r,\tau) \in R^{d+1}\})$$

$$Z_{XY} = \int D\varphi \; e^{-\int d\vec r\, d\tau \, (\partial_\tau \varphi)^2 + (\nabla\varphi)^2}, \quad \varphi \in [0, 2\pi]$$

$$\Rightarrow Z = \int D\phi \; e^{-\int d\vec r\, d\tau \, |\partial_\tau \phi|^2 + |\nabla \phi|^2} \delta(|\phi|^2 - |\phi_0|^2)$$

$$\simeq \int D\phi \; e^{-\int d\vec r\, d\tau \{ |\partial_\tau\phi|^2 + |\nabla\phi|^2 \}} \cdot e^{-\lambda(|\phi|^2 - |\phi_0^2|)^2}$$

$$[\phi(\vec r, \tau) \in \mathbb{C}]$$

$$\lambda > 0 \, , \; -2\lambda|\phi_0^2| = m^2 < 0$$

λ shall be large and positive ; m^2 negative.

$$Z = \int D\phi \; e^{-\int d\vec{r}\, d\tau \left[|\partial_\tau \phi|^2 + |\nabla\phi|^2 + m^2|\phi|^2 + \lambda|\phi|^4 \cdots \right]}$$