Phys525:
Quantum Condensed Matter Physics: Quantum Criticality
Basics, Dynamics and Topological criticality

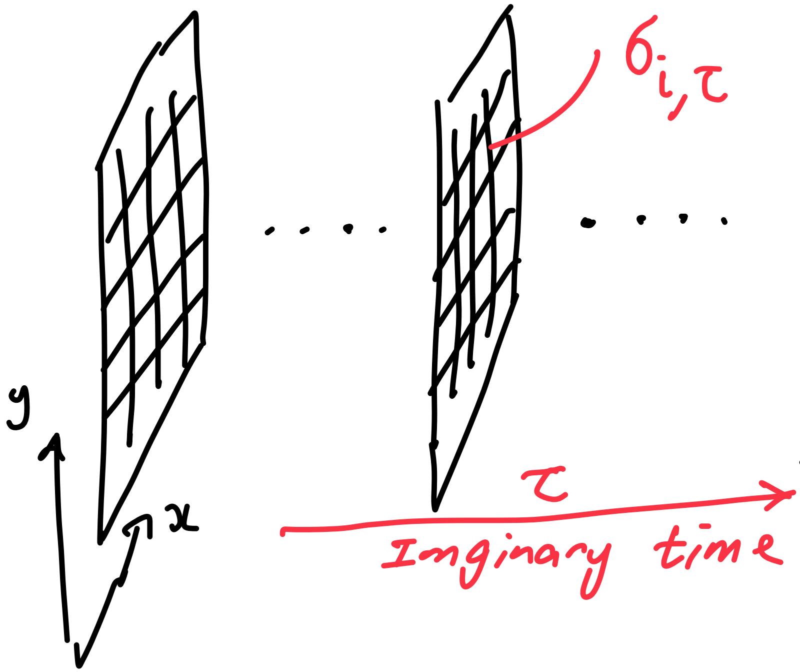
Episode 10:

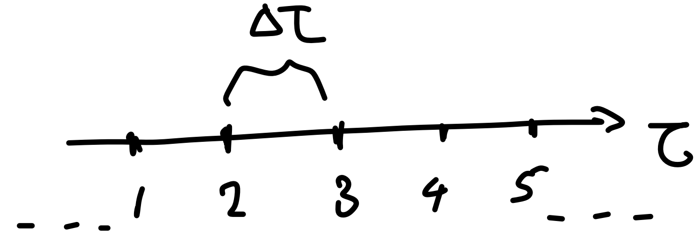
Imaginary time evolution——slightly more technical stuff

Two important ideas in QFT/EFT approaches

- Imaginary time evaluation of a quantum problem in d-dimensions=*partition-like functions* in d+1 dimensions (to identify space time symmetries in the QFT representations.)

- Coarse graining approach to lattice Models with discrete or continuous fields (it is used in both classical SM and quantum many-body physics. In many quantum problems, formally and technically constructing via Callan-Symanzik RGE approach).





$$H_{Ising} = -J \sum_{\langle ij \rangle} \sigma_{iz} \sigma_{jz} - I \sum_i \sigma_{ix}$$

Quantum Ising Spins

d-dimension

$$\tilde{J}(J\Delta\tau) = J\Delta\tau, \quad \tilde{K}(I\Delta\tau) = -\frac{1}{2} \ln \tanh(I\Delta\tau)$$

$$\beta \mathcal{H}_{Ising} = -\tilde{J}(J\Delta\tau) \sum_{\langle ij \rangle} \sigma_{i,\tau} \sigma_{j,\tau} - \tilde{K}(I\Delta\tau) \sum_{\langle \tau\tau' \rangle} \sigma_{i,\tau} \cdot \sigma_{i,\tau'}$$

Ising Model in (d+1)-dimension

d+1 dimension Euclidean space QFT

$$Z = \int D\phi \exp[-S(\{\phi(\mathbf{r},\tau)\})],$$

$$S = \int d\mathbf{r} d\tau [(\partial_\tau \phi)^2 + \nabla\phi \cdot \nabla\phi + m^2\phi^2 + \lambda\phi^4 + \ldots.]$$

- Space-time symmetry group is SO(d+1) (which is isomorphic to SO(d,1) group, the Minkowski space-time group.).

- Wilson-Fisher infrared fixed points etc follow. TBD.

imaginary time evolution (QFT: Vac-Vac Amplitude)

- Extract ground state properties (today)
- Extract space-time symmetries suitable for EFTs (today)
- Evaluate the dynamic correlations (later discussions)