Phys525:
Quantum Condensed Matter Physics: Quantum Criticality
Basics, Dynamics and Topological criticality

Episode 8:
Coarse graining and Imaginary time evolution

Coarse graining (classical Ising spins)

$$Z = \sum_{\{\sigma_i\}, i \in lattice} \exp(-\beta H_{Ising}(\{\sigma_i\}), \sigma_i = \pm 1$$

- Mapped into

$$Z = \int D\phi(r) exp(-F(\{\phi(r)\}), \int D\phi(r) = \Pi_r \int d\phi(r), \phi(r) \in R^1$$

$$F(\{\phi(\vec{r})\}) = \vec{\nabla}\phi(\vec{r}) \cdot \vec{\nabla}\phi(\vec{r})$$

$$+ \alpha \phi^2(\vec{r}) + \beta \phi^4(\vec{r}) + \cdots$$

* terms forbidden : $\vec{\nabla}, \vec{\nabla}^3, \cdots$

$\phi^3, \phi^5, \cdots$

* iRRelavent terms : $\phi^6, (\nabla^2\phi)^2, \cdots$

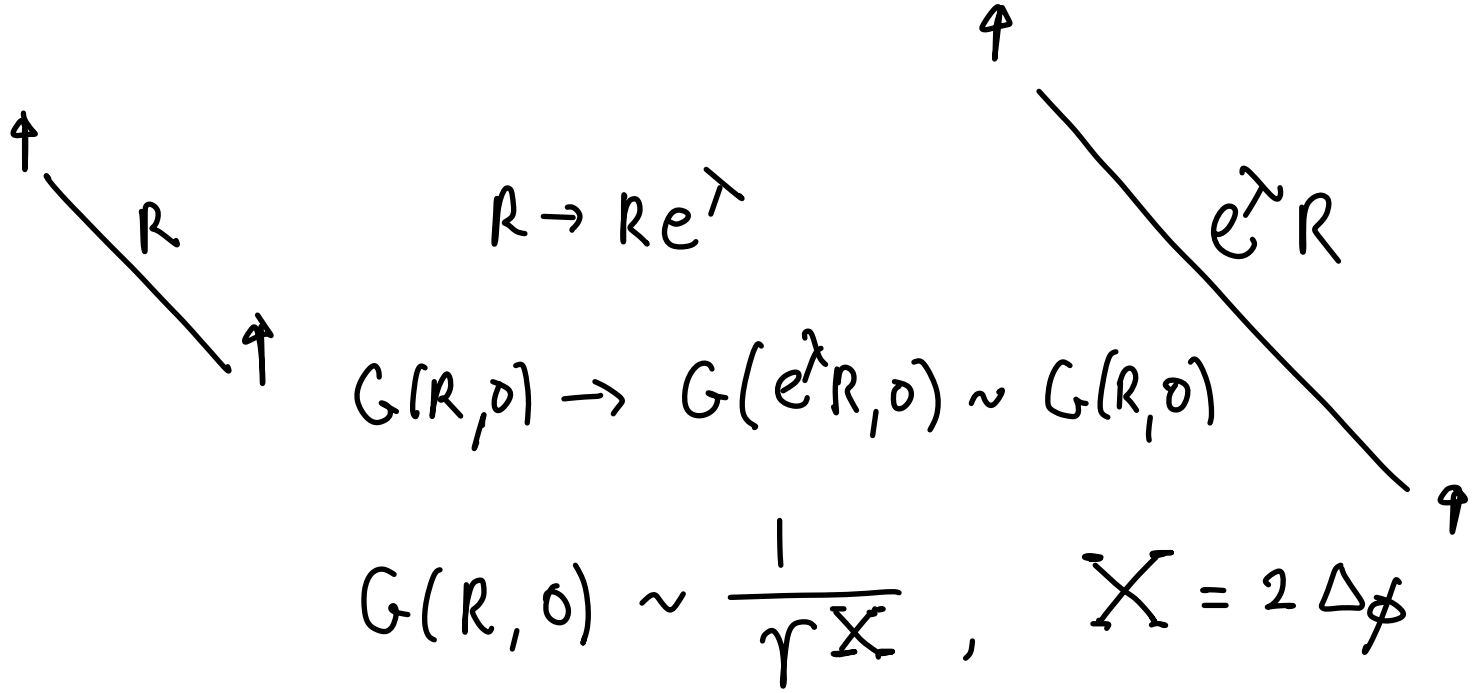
Remark on Scaling / RGE (More discussions later)

$$\tilde{F}_{\lambda} = \tilde{F}(\{\phi_i\}, i=1\cdots\infty; \overset{\sim}{\alpha}_\lambda, \overset{\sim}{\beta}_\lambda, \cdots)$$

for Grain size $N = 2^{\lambda}$ spins ; λ, scale factor.

$$\frac{d\overset{\sim}{\alpha}_\lambda}{d\lambda} = \beta_\alpha(\overset{\sim}{\alpha}_\lambda, \overset{\sim}{\beta}_\lambda), \quad \frac{d\overset{\sim}{\beta}_\lambda}{d\lambda} = \beta_\beta(\overset{\sim}{\alpha}_\lambda, \overset{\sim}{\beta}_\lambda)$$

Supplementary materials on scale symmetry (a very practical definition)



$$G(e^{\lambda}R,0) = e^{-\lambda \cdot 2\Delta_{\phi}} G(R,0)$$

$$\vec{r} \rightarrow \vec{r}\,' = \frac{\vec{r}}{e^{\lambda}} \quad \text{or} \quad \vec{r} = e^{\lambda}\vec{r}\,'$$

$$\psi(\vec{r}) \rightarrow \psi'(\vec{r}\,') = e^{\lambda \Delta_\phi}\psi(\vec{r})$$

$$G'(\vec{r}\,', 0) = e^{2\lambda\Delta_\phi} G(\vec{r}, 0)$$

↑ Rescaled

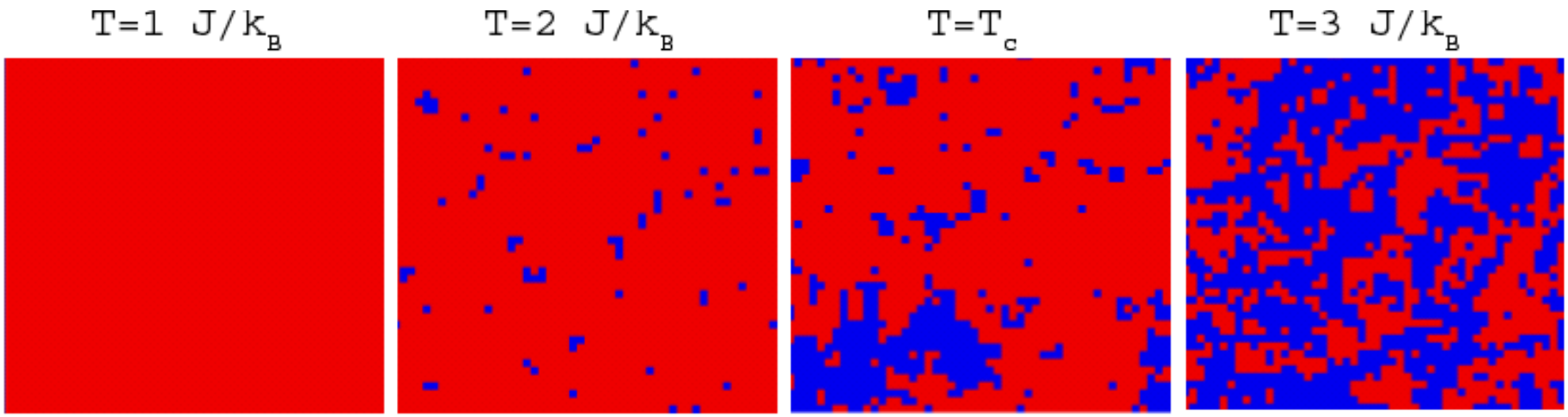
$$= G(\vec{r}\,', 0)$$

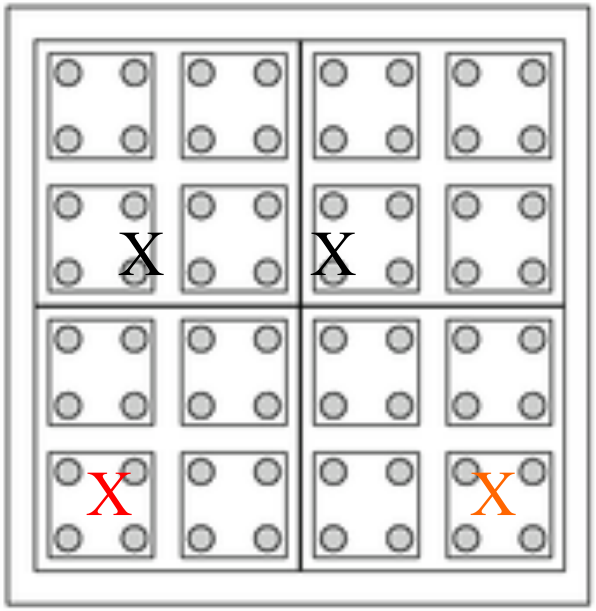
Definition of Scale Symmetry via fields

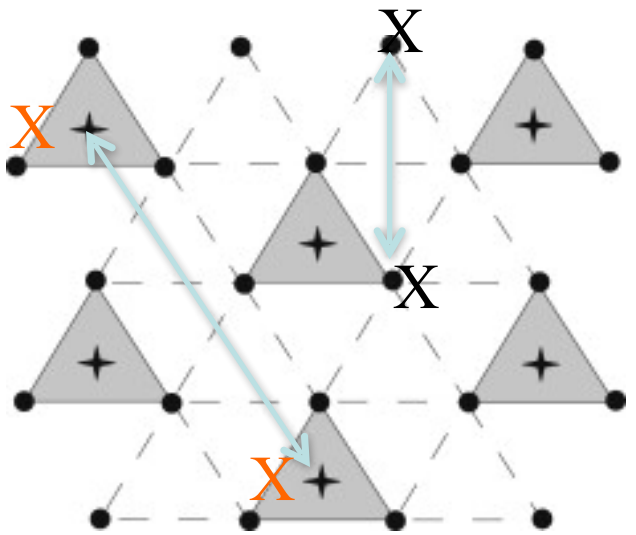
Early works in SM: Kadanoff block spins (60s)

- *Coarse graining* without using real fields but using a **block spin** to represent dynamics of a grain of a few spins and then repeat many times.

Emergent Scale Symmetry at critical point of Ising Model (H= -J \sum_{ij} S_i *S_j, Kadanoff, 1960s)





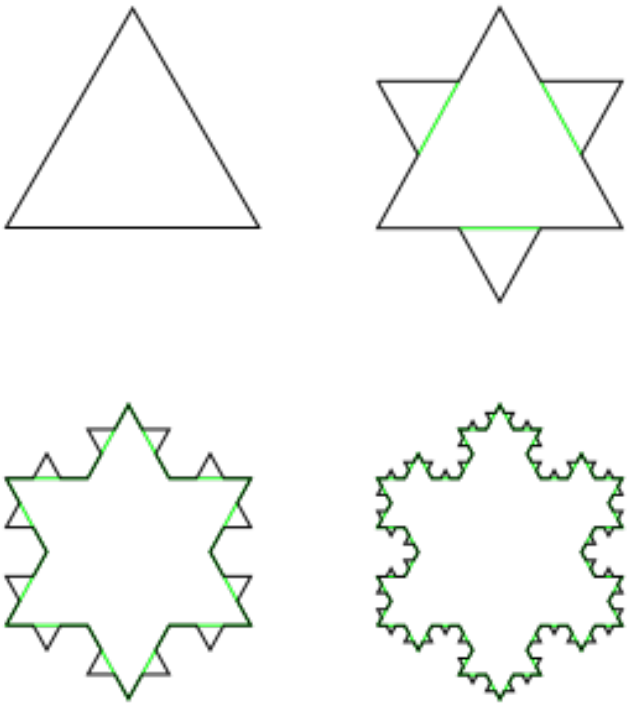


X: spin; X: block spin

Scale symmetries in nature

The correlation of BLOCK spins (b X b) at distance bL away
= The the correlations of Microscopic spins at distance L away.

-->The correlation of coarse grained fields at distance Lb
=The correlation of original fields at distance L



Strongly Interacting Physics in Condensed Matter Physics: "Emergent scale symmetries"

40s-50s	60s-70s	80s-now
…	…	…
Fermi liquids;	Scale symmetry at critical points, Wilsonian renormalization.	HTcS;
Order parameter / phase transitions;	Topological Phase transitions (Kosterliz-Thouless)	FQH;
BCS superconductivity	…	Topological/non-topological spin states ("topologically Order parameters"?) (Kitaev, Preskill, 2006; Levin,Wen, 2006)
…		

Back to the quantum Ising model.