Phys525:
Quantum Condensed Matter Physics: Quantum Criticality
Basics, Dynamics and Topological criticality

Episode 6:

Two important ideas in EFT approach to emergent phenomena in interacting physics

Type of emergent symmetries

- A) space-time symmetries (Either due to crystal structures or particle-particle interactions);

- B1) internal ES such as PHC (either due to crystal structures or via interactions that lead to SSB);

- B2) “gauged symmetries”—-Emergent gauge fields, typically in topologically ordered spin liquids with anyons.

Why shall we care

- All correlations depend on emergent space-time symmetries and hence quantum dynamics.

- Universality of strong coupling physics near Fixed points crucially depend on both internal ES and space-time ESs.

- Topological states/topologies depend on internal ES.

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To identify the QFT/EFT for a quantum system

- Identify all the internal symmetries and find a field representation; Examples: Z_2 Ising, U(1) charge, SU(2) spin rotation, PHC, TRS etc.

- Identify the space-time symmetries; Example: Galilean symmetry(z=2), Lorentz symmetry (z=1), scale symmetry and/or Conformal symmetry (Non-relativistic CFT vs relativistic ones).

- Remark: In the most interesting cases, EFT depends on representations of the symmetry group. I.e. depends on the Young's tableau of the representation. see SU(N) spin chain discussions in reading materials set 3.

Two important ideas in QFT/EFT approaches

- Imaginary time evaluation of a quantum problem in d-dimensions=partition functions in d+1 dimensions (to identify space time symmetries in the QFT representations.)

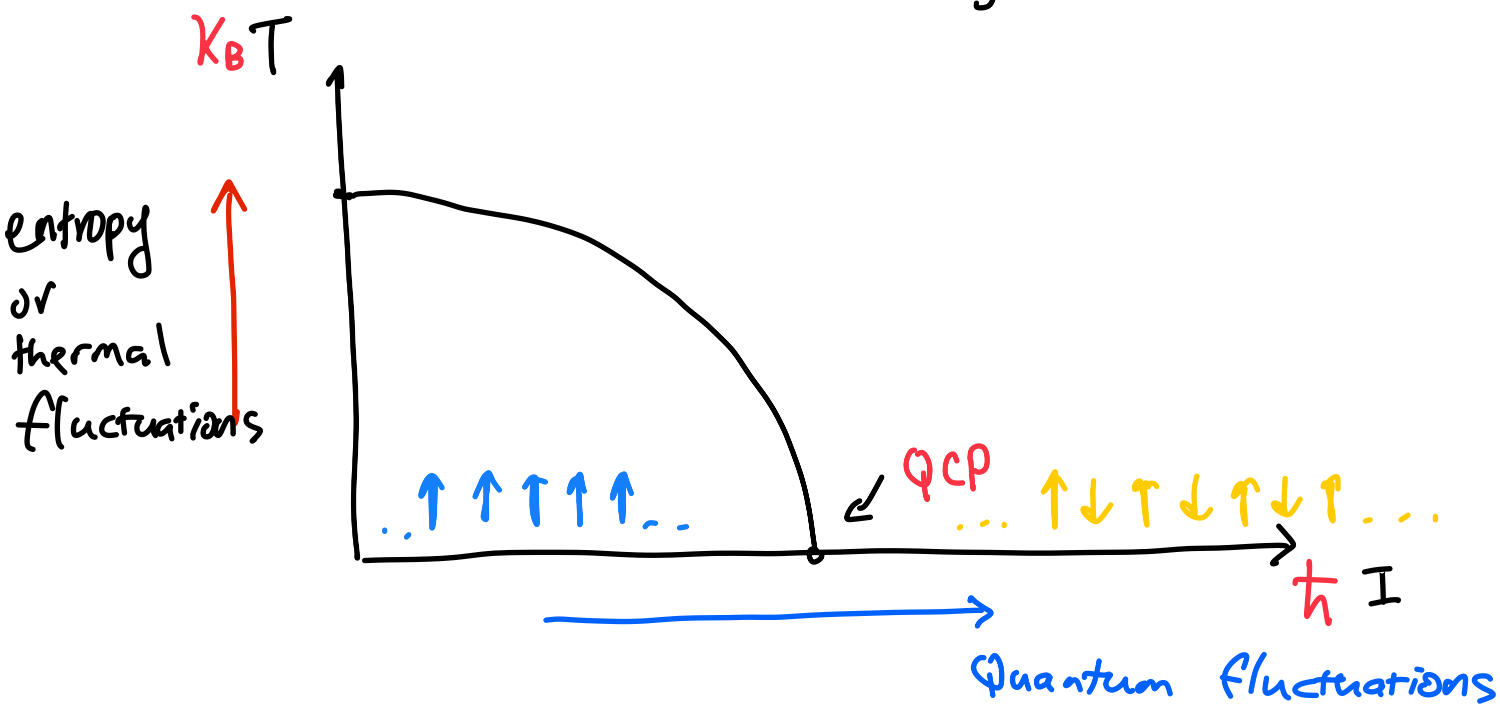
- Coarse graining approach to lattice Models with discrete or continuous fields (it is used in both classical SM and quantum many-body physics. In many quantum problems, formally and technically constructing via Callan-Symanzik RGE approach).

Towards Quantum Models

$$[S_{\alpha i}, S_{\beta j}] = i\hbar\, \epsilon_{\alpha\beta\gamma} S_{\gamma i}\, \delta_{ij}$$

$$\mathcal{H}_{Ising} = -J \sum_{\langle ij \rangle} S_{z_i} \cdot S_{z_j} + I \sum_i S_{x_i}$$

$$[S_{z_i}, H_{Ising}] = i\hbar\, I \sum_{i \in j} S_{y_i}$$



Towards Quantum Models

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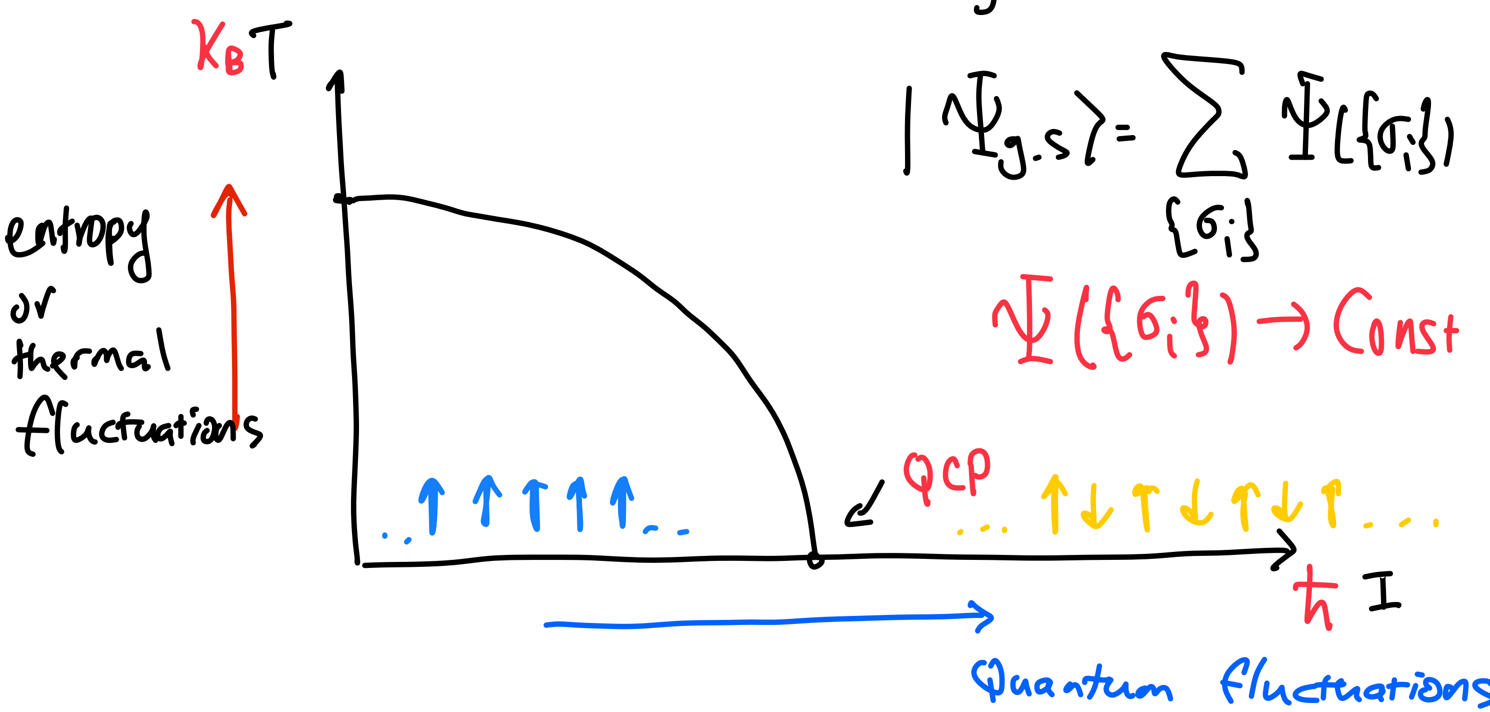
$$[S_{z_i}, H_{Ising}] = i\hbar\, I \sum_{i \in j} S_{y_i}$$

$$|\Psi_{g.s}\rangle = \sum_{\{\sigma_i\}} \Psi(\{\sigma_i\})\, |\sigma_1, \sigma_2, \dots \sigma_N\rangle$$

$\Psi(\{\sigma_i\}) \to$ Const if $I \to \infty$



Review of *Coarse graining* in SM: a classical Ising field

- Ising field sigma=+1, -1 is equivalent to a real field \phi.

- Partition function is equivalent to a classical \phi-4 theory (I.e. the Wilson-Fisher approach to critical phenomena.)

The Concept of "<u>coarse graining</u>": an illustration

state : $\{\sigma_{z_i},\ i=1,2,\ldots\infty\}$

$$\ldots \uparrow\ \uparrow\ \uparrow\ \downarrow\ \uparrow\ \downarrow\ \uparrow\ \downarrow \ldots \longrightarrow Z = \sum_{\{\sigma_{z_i}\}} e^{-\beta\mathcal{H}(\{\sigma_{z_i}\})}$$

$\beta\mathcal{H}(\{\sigma_{z_i}\})$

thermal physics

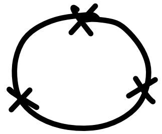
field : $\{\phi(\vec{r})\}$, $\phi(\vec{r}) = \langle \sigma_{z_i} \rangle_{\vec{r}}$

$$\ldots \underbrace{\uparrow\ \uparrow\ \uparrow\ \downarrow}_{\phi(\vec{r}_1)}\ \underbrace{\uparrow\ \downarrow\ \uparrow\ \downarrow}_{\phi(\vec{r}_2)} \ldots \longrightarrow Z = \sum_{\{\phi(\vec{r})\}} \underbrace{\sum_{\{\sigma_{z_i}\} \in \{\phi(\vec{r})\}} e^{-\beta H(\{\sigma_{z_i}\})}}$$

$\beta F(\{\phi(\vec{r})\})$

$$= \sum_{\{\phi(\vec{r})\}} e^{-\beta F(\{\phi(\vec{r})\})}$$

3-Spin Grain



A Cartoon

$\{\sigma_{z_i}\}$	$\mathcal{H}$	$\longrightarrow$ ϕ $= \langle \sigma_{z_i} \rangle$	$F(\phi)$ $= U - TS$
$\uparrow\uparrow\uparrow$	$-3J$	$+1$	$-3J$
$\uparrow\uparrow\downarrow$	J		
$\uparrow\downarrow\uparrow$	J	$+\frac{1}{3}$	$J - T\ln 3$
$\downarrow\uparrow\uparrow$	J		
$\uparrow\downarrow\downarrow$	J		
$\downarrow\uparrow\downarrow$	J	$-\frac{1}{3}$	$J - T\ln 3$
$\downarrow\downarrow\uparrow$	J		
$\downarrow\downarrow\downarrow$	$-3J$	-1	$-3J$

$$Z = \sum_{\{\sigma_{z_i}\}} e^{-\beta H} \longrightarrow Z = \sum_{\{\phi(\vec{r})\}} e^{-\beta F(\phi)}$$

N-Spin Grain (with $N \to \infty$) Microensemble

$$\phi(M) = \frac{1}{N} \sum_{i=1}^{N} \sigma_i ,$$

$\binom{N}{M}$ configurations with $\phi(M)$

$$= 1 - \frac{2M}{N} , \quad M = 0, \dots N \quad \text{(M down Spins)}$$

$$m \in \underbrace{\left[-1, -1 + \frac{2}{N}, \dots, 0, \dots, 1 - \frac{2}{N}, 1 \right]}_{N+1}$$

$$\text{~~total~~ Configurations} = \sum_{M=0}^{\infty} \binom{N}{M} = 2^N$$

$$F(\{\phi(\vec{r})\}) = \vec{\nabla}\phi(\vec{r}) \cdot \vec{\nabla}\phi(\vec{r})$$
$$+ \alpha\,\phi^2(\vec{r}) + \beta\,\phi^4(\vec{r}) + \cdots$$

* terms forbidden : $\vec{\nabla}$, $\vec{\nabla}^3$, ...

  ϕ^3, ϕ^5, ...

* iRRelavent terms : ϕ^6, $(\nabla^2\phi)^2$, ...