Phys525:
Quantum Condensed Matter Physics:

Episode 5: PHC as an internal emergent symmetries (ES):
summary of Haldane model(can also be relativistic) and PHC in SSB states

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Tentative schedule I (before the reading break of Feb 20-24, subjected to revisions)

Week one:
L1: general information on the course

L2: Why emergent symmetries/emergent space-time symmetries?
——perspectives from QCP points of view (I.e. space-time symmetries)
——perspectives from Topological point of view (I.e. internal symmetries)

Week two: Emergent space-time symmetries in Quantum Crystals

L3: Emergent Galilean symmetry near Lifshitz points

L4: Emergent Lorentz symmetry in lattices

Week three and four: Emergent space-time symmetries in strongly interacting systems (Two important ideas)

L5: Emergent symmetries in quantum interacting spin models:
From discrete quantum (or classical) lattice models to continuous field theories

A) the imaginary time evolution—-the Vac-Vac transition amplitude in Quantum models;
B) the method of Coarse graining spins.

L6: Emergent symmetries in quantum interacting particle models I

A) coarse graining quantum particles;
B) emergent symmetries in Bose-Hubbard models: Galilean invariance vs Lorentz invariance.

L7: Open discussions: Emergent symmetries in quantum interacting particle models II

A) Emergent locality and causality.

L8: Open discussions: Emergent symmetries in quantum interacting particle models III

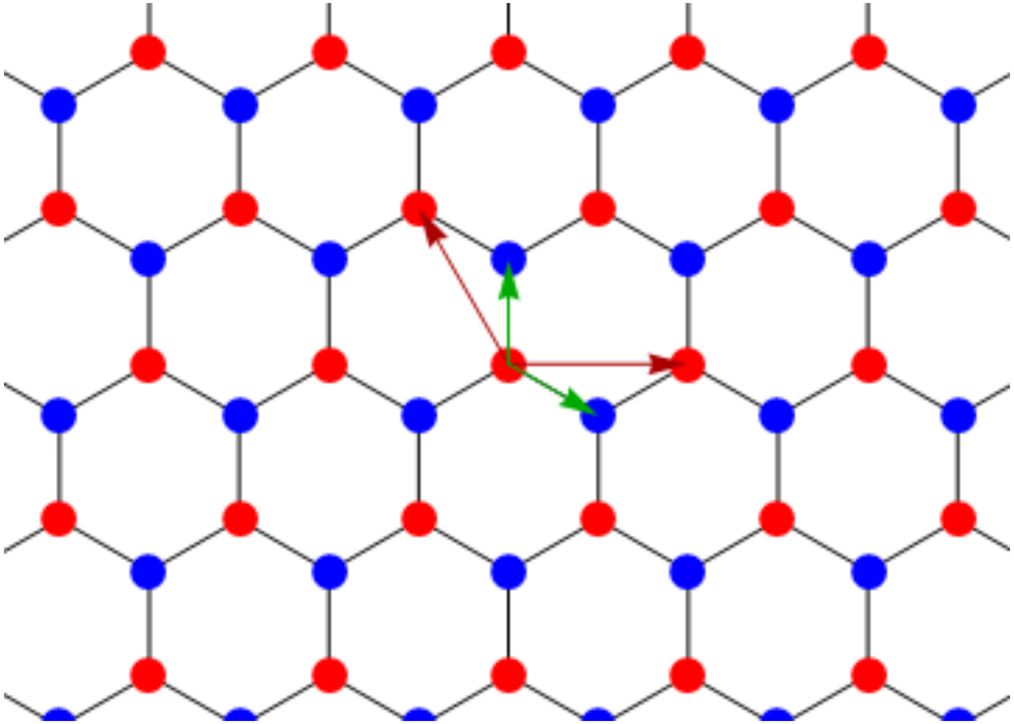
A) Quantum dynamics of real fermions;
B) Emergent Lorentz symmetries in real fermions.

Week five and six: Emergent Scale and conformal symmetries in strongly interacting systems

L9: Scale symmetries and basic ideas of Wilsonian approaches to scale symmetries

L10: How it works? Ideas of fixed points and anomalous dimensions

Discussions on a more realistic Haldane model in honeycomb lattices

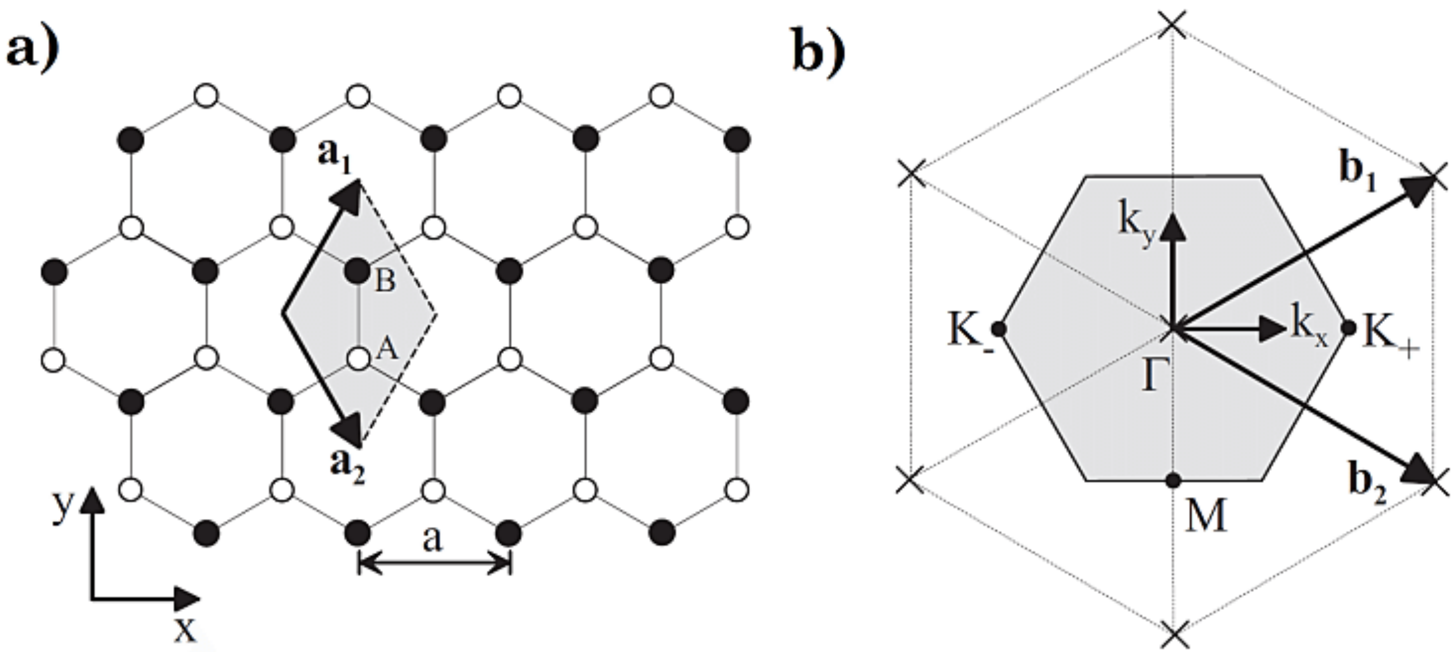


- NN hoping only and no internal B field lead to an **emergent** PHC/CC symmetry in addition to TRS. Note that PHC is not a fundamental/ generic symmetry of the honeycomb lattice.

PHC/CC, TRS

$$\mathcal{H}(p, m) = \tau_z \otimes \sigma_x p_x + \sigma_y p_y + m \sigma_z$$

τ_z : Valley Subspace ; $\sigma_{x,y,z}$: Sublattice Subspace



PHC Transformation:

$$C = K \, \mathbb{1} \otimes \sigma_x$$

K → complex Conjugate

$$C^{\dagger} \mathcal{H}(p, m) \, C = -\mathcal{H}(-p, m)$$

Time Reversal Transformation:

$$\mathcal{T} = K \tau_x \, , \quad \mathcal{T}^{\dagger} \mathcal{H}(p.m) \, \mathcal{T} = +\mathcal{H}(-p, m)$$

Chiral Transformation

$$S = \tau_x \otimes \sigma_x \quad S^{\dagger} \mathcal{H}(p.m) \, \mathcal{T} = -\mathcal{H}(p.m)$$

C is not unique because of additional U(1) "chiral" symmetry

$$\mathcal{U}_A^{\dagger} \mathcal{H} \mathcal{U}_A = \mathcal{H}, \quad \mathcal{U}_A^{\dagger} \mathcal{U}_A = \mathbb{1}$$

then CC Tranformation is defined up to $\mathcal{U}_A$.

i.e. if $C^{\dagger} \mathcal{H} C = -\mathcal{H}$, then for $\mathcal{U}_A^{\dagger} C \mathcal{U}_A = C'$

$$C'^{\dagger} \mathcal{H} C' = -\mathcal{H}$$

More explicitly,

$$C' = \mathcal{U}_A^{\dagger} K \mathcal{U}_c \mathcal{U}_A = K \mathcal{U}_A^{T} \mathcal{U}_c \mathcal{U}_A$$

In the Haldane Model, C defined up to u_A^2.

$$u_A = e^{i\tau_z \theta/2}, \qquad C = K \mathbb{1} \otimes \sigma_x$$

$$C' = u_A^{+} K \mathbb{1} \otimes \sigma_x u_A = K u_A^{T} \mathbb{1} \otimes \sigma_x u_A$$

$$= K \mathbb{1} \otimes \sigma_x u_A^2, \qquad u_A^T = u_A$$

$$= C u_A^2 = C \cdot [\cos\theta + i \sin\theta \, \tau_z]$$

Different limits of the Haldane lattice Model

- With only NN hoping and without magnetic fields, it has PHC/CC symmetry and also TRS. (Belongs to BDI, chiral orthogonal class). No QHE.

- Breaking TRS (via NNN hoping in the model+ internal magnetic fields) in this particular Haldane's proposal, it also breaks PHC/CC symmetry. Leads QHE. (Belongs to A, unitary class).

		TRS	PHS	SLS	$d=1$	$d=2$	$d=3$
standard (Wigner-Dyson)	A (unitary)	0	0	0	-	$\mathbb{Z}$	-
	AI (orthogonal)	$+1$	0	0	-	-	-
	AII (symplectic)	-1	0	0	-	$\mathbb{Z}_2$	$\mathbb{Z}_2$
chiral (sublattice)	AIII (chiral unitary)	0	0	1	$\mathbb{Z}$	-	$\mathbb{Z}$
	BDI (chiral orthogonal)	$+1$	$+1$	1	$\mathbb{Z}$	-	-
	CII (chiral symplectic)	-1	-1	1	$\mathbb{Z}$	-	$\mathbb{Z}_2$
BdG	D	0	$+1$	0	$\mathbb{Z}_2$	$\mathbb{Z}$	-
	C	0	-1	0	-	$\mathbb{Z}$	-
	DIII	-1	$+1$	1	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$
	CI	$+1$	-1	1	-	-	$\mathbb{Z}$

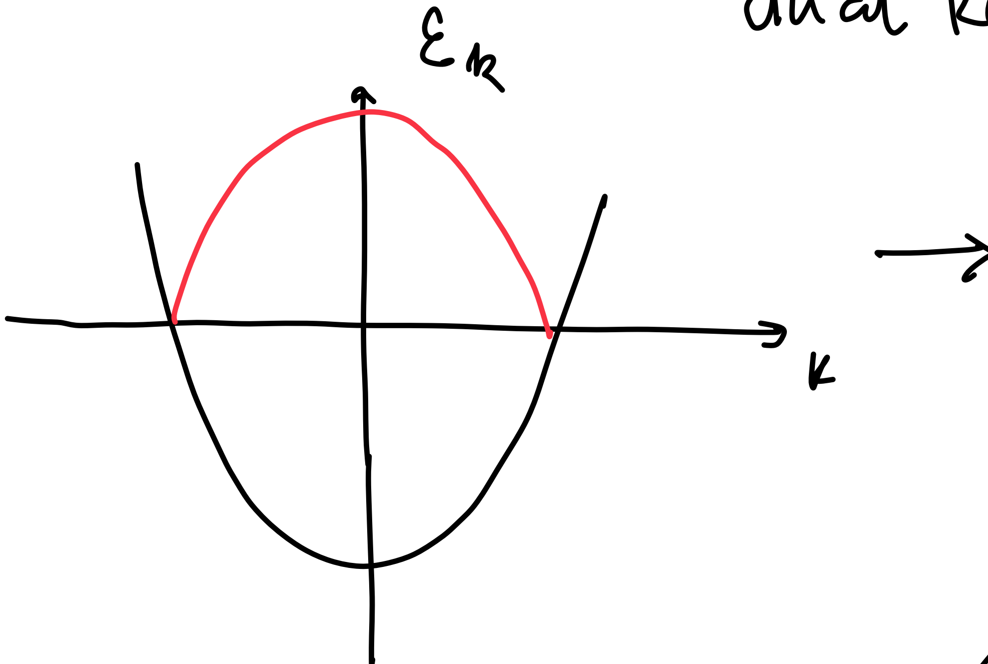
metry classes of single particle Hamiltonians classified in terms of the presence or abse
d particle-hole symmetry (PHS), as well as sublattice (or "chiral") symmetry (SLS).[36]
ies is denoted by "0". The presence of these symmetries is denoted either by "+1" c
iunitary) operator implementing the symmetry at the level of the single-particle Ham
ext). [The index ± 1 equals η_c in Eq. (1b); here $\epsilon_c = +1, -1$ for TRS and PHS, respect
BLE (which can be realized in non-superconducting systems) TRS $= +1$ when the S
in the text] and TRS $= -1$ when it is a half-integer [called TRS (odd) in the text]
nductor "Bogoliubov-de Gennes" (BdG) symmetry classes D, C, DIII, and CI, the Ha
tion symmetry when PHS$=-1$ [called PHS (singlet) in the text], while it does not pı
S (triplet) in the text]. The last three columns list all topologically non-trivial quantu
etry class and spatial dimension. The symbols $\mathbb{Z}$ and $\mathbb{Z}_2$ indicate whether the space
into topological sectors labeled by an integer or a $\mathbb{Z}_2$ quantity, respectively.

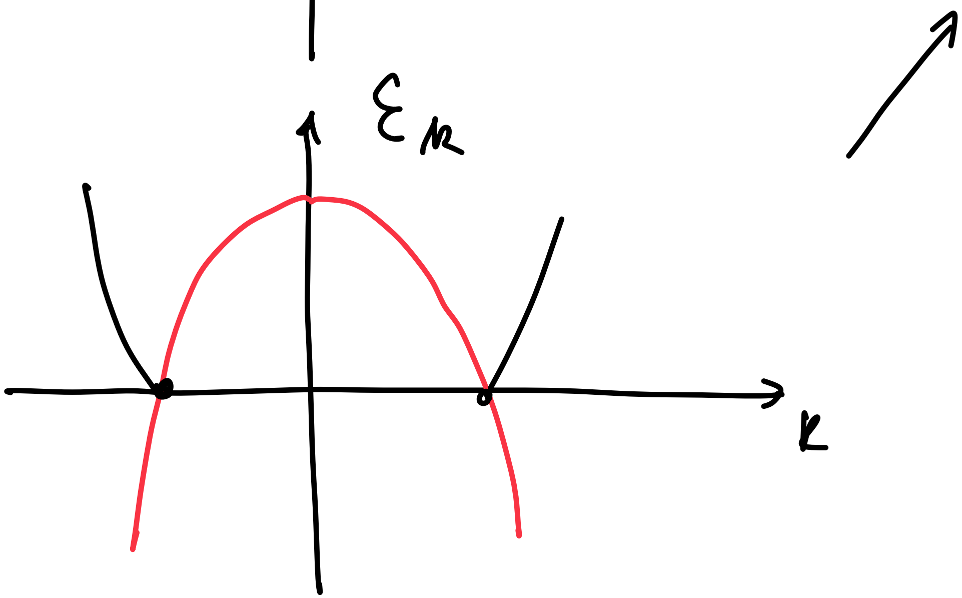
Today's plan

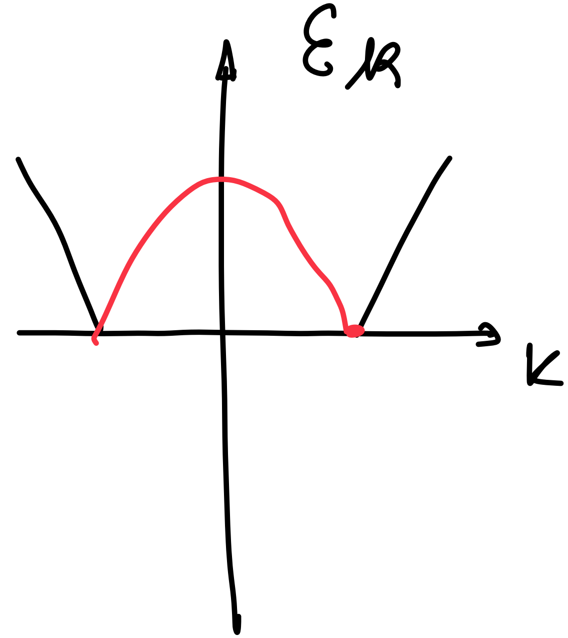
- PHC/CC symmetry in SSB states (particle-particle interaction driven)

more interesting case: PHC symmetry without U(1) symmetry via SSB

dual Reps



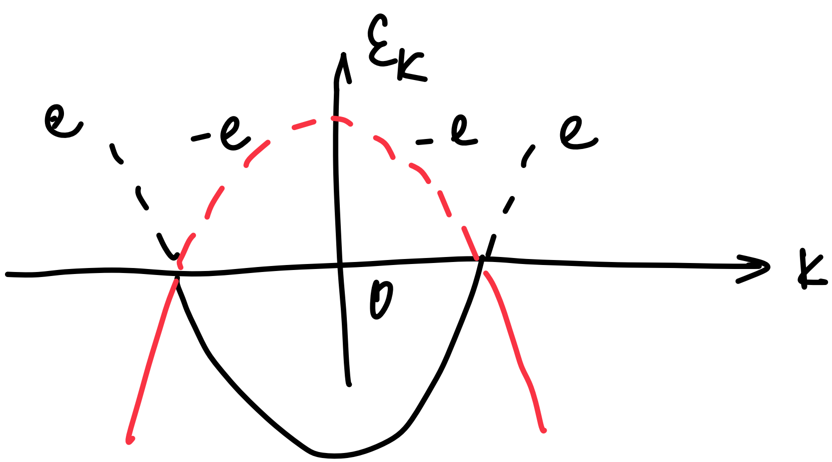


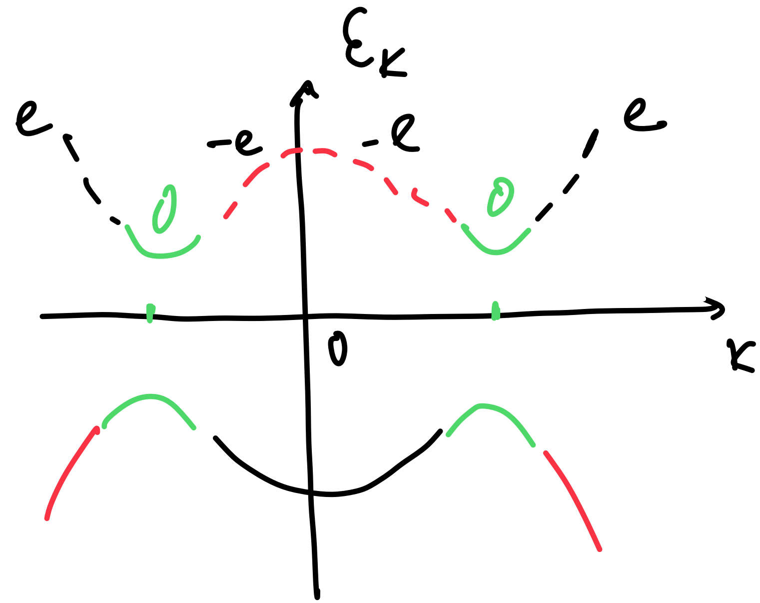


Examples of interaction-induced EBs

$$H = H_0 + g\int \psi^\dagger \sigma_y (\psi^\dagger)^T \psi^T \sigma_y \psi$$

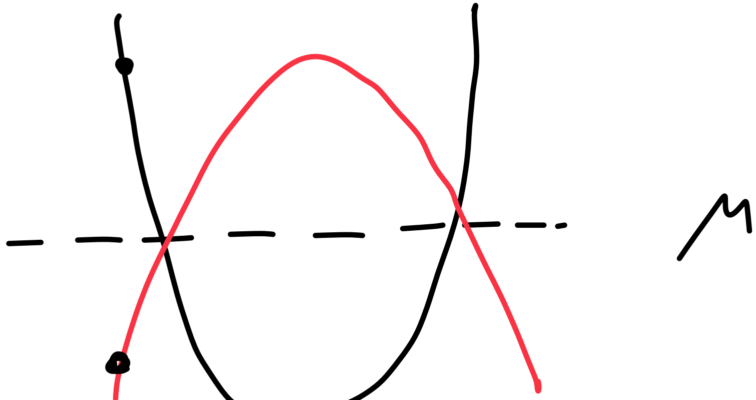
$\psi^\dagger_{k,\uparrow}|BCS\rangle \simeq \psi_{-k,\downarrow}|BCS\rangle$

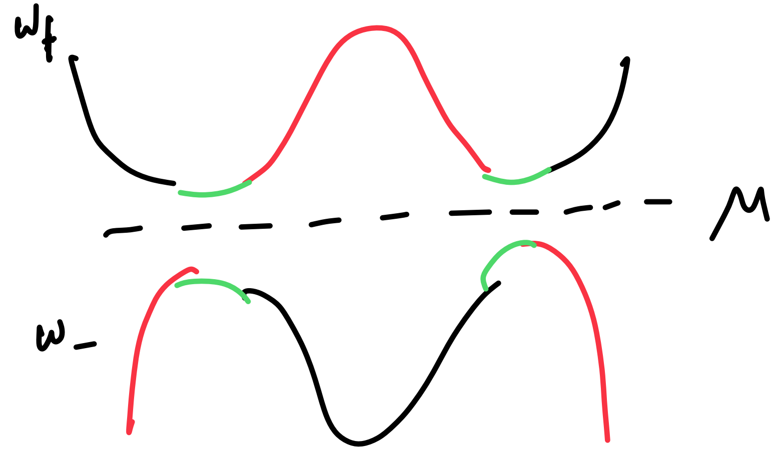




PHC Symmetry
(Real fermion Rep.)

for any k $\quad \psi^{\dagger}_{k\uparrow} |BCS\rangle \leftrightarrow \psi_{-k\downarrow} |BCS\rangle$





$$\mathcal{H}_k = \begin{bmatrix} \varepsilon_k + \mu & \hat{\Delta} \\ \hat{\Delta} & \mu - \varepsilon_{+k} \end{bmatrix}$$

$$\gamma^{\dagger}_{(+)\, k\uparrow} = \gamma_{(-)\, -k\downarrow}$$

PHC

$$H = \sum_k \Psi^{\dagger}_k \mathcal{H}_k \Psi_k$$

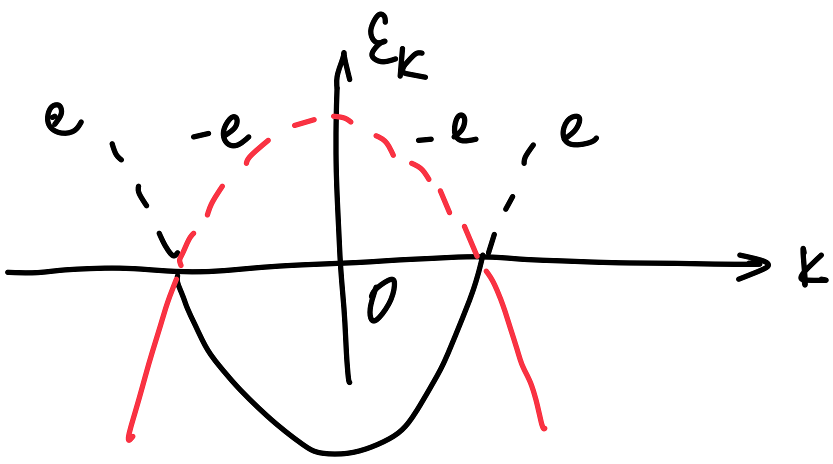
Ψ_k – Nambu spinor

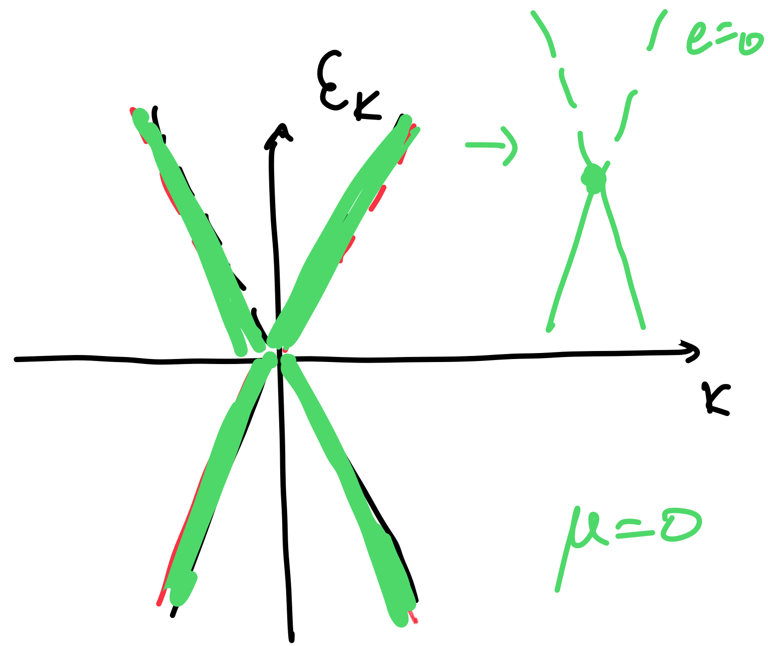
Examples of interaction-induced EBs

$$H = H_0 + g\int \psi^\dagger \nabla \psi^\dagger \psi \nabla \psi$$

$(g = g_c^*)$

$\psi_k^\dagger |BCS\rangle \simeq \psi_{-k} |BCS\rangle$





"PHC Symmetry"

Real fermion excitations

(Read & Green, 00)

- Emergent PHC/CC symmetries: case analysis
- A) due to interactions with background crystal structures, i.e. band structures with U(1) charge symmetries. (Often need tuning!!)
- B) due to mutual interactions between particles via spontaneous symmetry breaking. (Very robust!!)