Phys525:
Quantum Condensed Matter Physics:

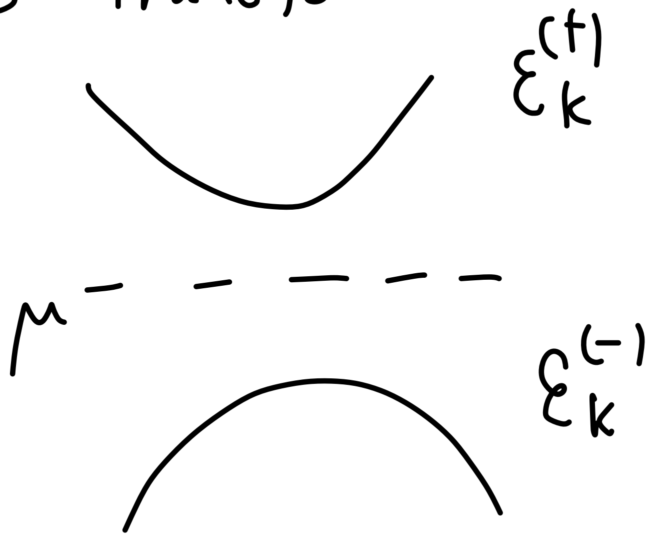
Episode 4: PHC as an internal emergent symmetries (ES):
slightly formal stuff and PHC in SSB states

- Emergent PHC/CC symmetries: case analysis
- A) due to interactions with background crystal structures, i.e. band structures with U(1) charge symmetries. (Often need tuning!!)
- B) due to mutual interactions between particles via spontaneous symmetry breaking. (Very robust!!)

Today’s plan

- Some more formal/general discussions on PH transformations and basic algebraic structures.
- More realistic Haldane lattice model
- PHC in SSB states (particle-particle interaction driven)

C-Transformation



$b^\dagger_{k,+} \to b_{-k,-}$

$b_{k,+} \to b^\dagger_{-k,-}$

$H \to H$ if $\varepsilon^{(+)}_k = -\varepsilon^{(-)}_{-k}$

More formally, $C^\dagger \mathcal{H}(p,m)\, C = -\mathcal{H}(-p,m)$

$$H = \sum_k \varepsilon^{(+)}_k b^\dagger_{k,+} b_{k,+} + \varepsilon^{(-)}_k b^\dagger_{k,-} b_{k,-}$$

$$\left[= \sum_k \varepsilon^{(+)}_k b^\dagger_{k,+} b_{k,+} - \varepsilon^{(-)}_{-k} \overbrace{b_{-k,-} b^\dagger_{-k,-}}^{\to\, a^\dagger_{k,+} a_{k,+}} \right]$$

- More technical discussions on PHC transformation and PHC symmetry as supplementary materials.

(A)

$$H = \sum_{ij} \psi_i^\dagger \mathcal{H}_{ij} \psi_j = \Psi^\dagger \mathcal{H} \Psi$$

$$\Psi \to [\Psi'^\dagger]^T = [\Psi^\dagger U^\dagger]^T, \quad \Psi^\dagger \to \Psi'^T = \Psi^T U^T$$

$$H \Rightarrow \Psi^T U^T \mathcal{H} U^{\dagger T} \Psi^{\dagger T} \qquad \boxed{\mathrm{Tr}\, AB = \mathrm{Tr}\, B^T A^T}$$

$$= -\Psi^\dagger \underbrace{U^\dagger \mathcal{H}^T U}_{\mathcal{H}'} \Psi,$$

Thus

$$-U^\dagger \mathcal{H}^T U = \mathcal{H} = -\overbrace{U^\dagger K}^{C^\dagger} \mathcal{H} \overbrace{K U}^{C}$$

then H Remains invariant.

(B) $\Psi_i \longrightarrow \Psi_i'^{\dagger} = U^{\dagger}_{ji}\Psi^{\dagger}_j$, $\Psi^{\dagger}_i \to \Psi'_i = U_{ij}\Psi_j$

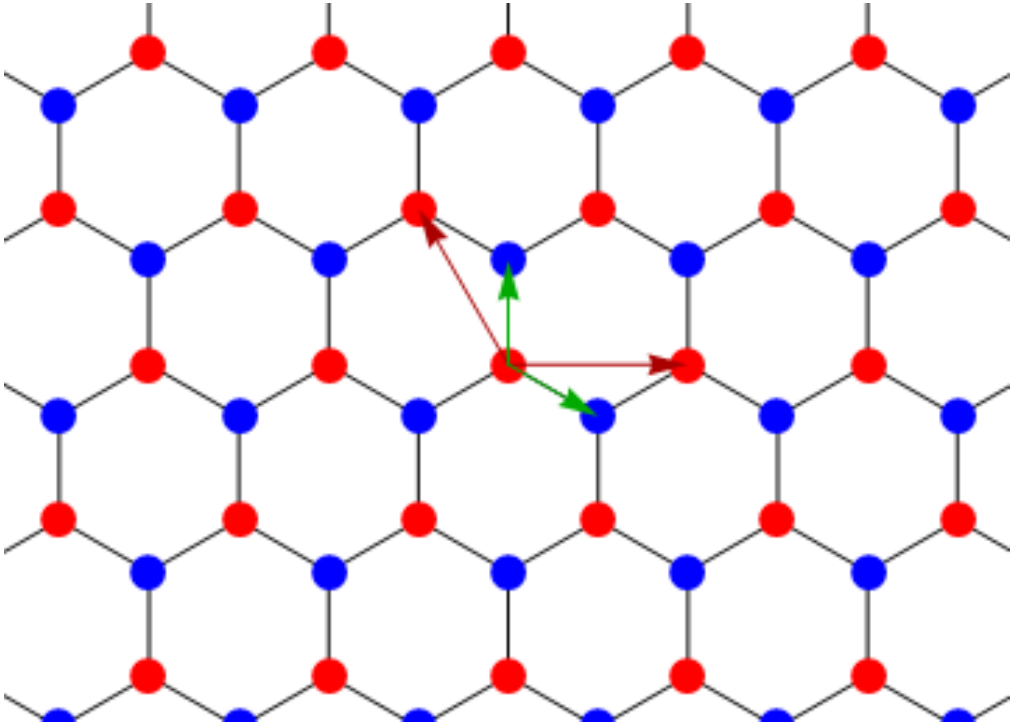
$$H = \sum_{ij} \Psi^{\dagger}_i \mathcal{H}_{ij} \Psi_j = \sum_{ij} \psi_\alpha \underbrace{U_{i\alpha} \mathcal{H}_{ij} U^{\dagger}_{\beta j}}_{U^{\dagger}_{\beta j} \mathcal{H}^{*}_{ji} U_{i\alpha}} \Psi^{\dagger}_\beta$$

$$= + \sum_{\alpha\beta} \psi^{\dagger}_\beta \mathcal{H}'_{\beta\alpha} \psi_\alpha, \quad \mathcal{H}'_{\beta\alpha} = -U^{\dagger} K \mathcal{H} K U$$

$$\mathcal{H} = -C^{\dagger} \mathcal{H} C \quad \text{or} \quad -\mathcal{H}(-p, m) = C^{\dagger} \mathcal{H}(p, m) C$$

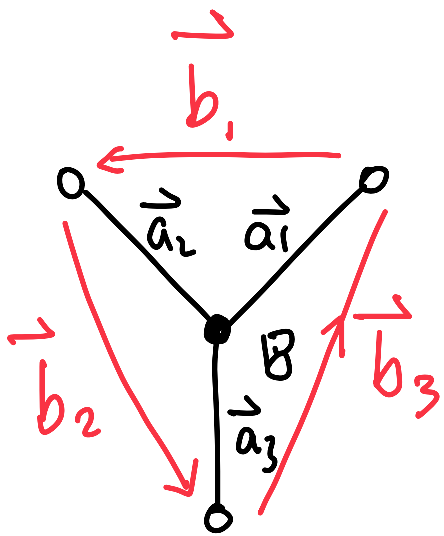
if H is invariant.

Discussions on a more realistic Haldane model in honeycomb lattices



$$\mathcal{H} = [\psi_A^+, \psi_B^+] \begin{bmatrix} 0 & t_{ab} \\ t_{ba} & 0 \end{bmatrix} \begin{bmatrix} \psi_A(k) \\ \psi_B(k) \end{bmatrix} + \cdots$$

$$t_{AB} = t_{BA}^* = \sum_{\alpha=1,2,3} e^{i \vec{k} \cdot \vec{a}_\alpha}$$

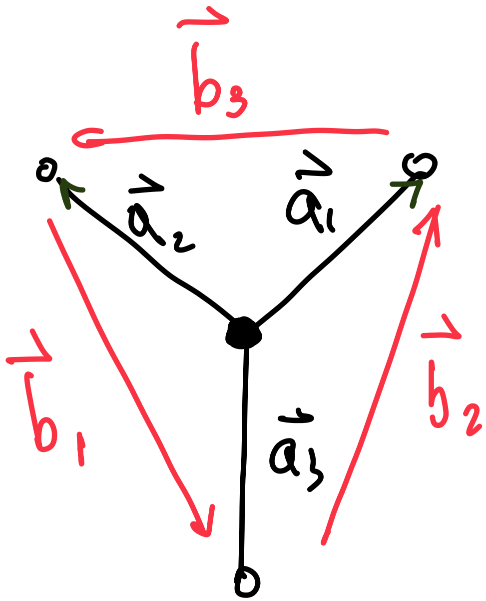


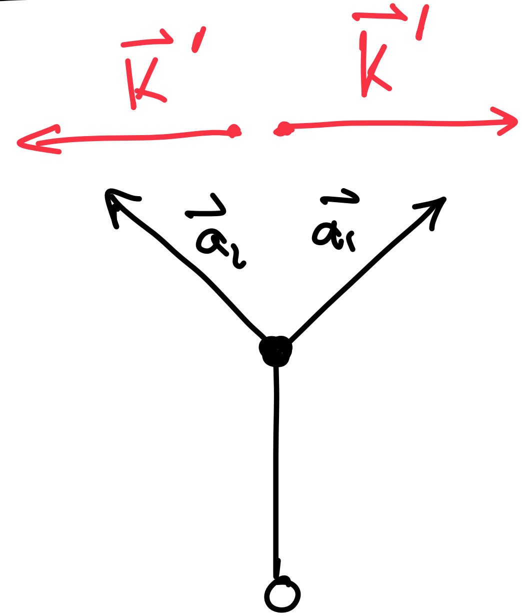
if $t_{AB}(\vec{k}) = 0$, then

$$(\vec{k}\cdot\vec{a}_1, \vec{k}\cdot\vec{a}_2, \vec{k}\cdot\vec{a}_3) = \left(0, \frac{2\pi}{3}, -\frac{2\pi}{3}\right)$$ per

$3! = 6$ solutions

There three pairs of Solutions



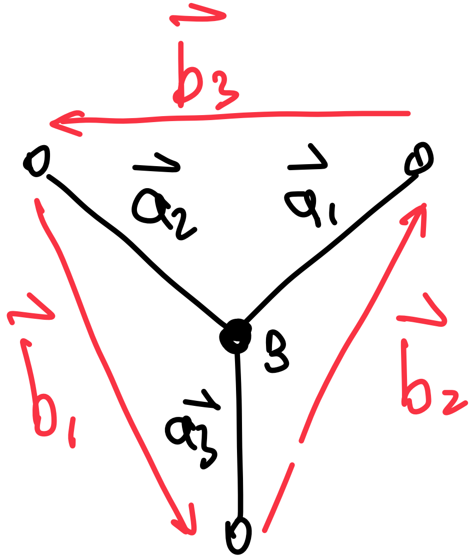


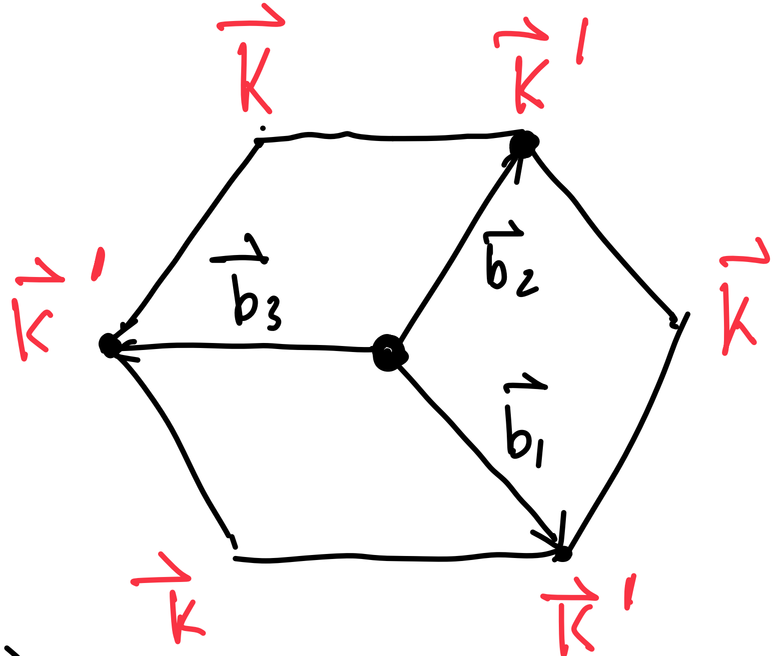
$$\vec{K} \longrightarrow \vec{K}' = -\vec{K}$$

$$(\vec{K}_1, -\vec{K}_1), (\vec{K}_2, -\vec{K}_2), (\vec{K}_3, -\vec{K}_3)$$

$$\left(0, \frac{2\pi}{3}, -\frac{2\pi}{3}\right) \longrightarrow \left(0, -\frac{2\pi}{3}, \frac{2\pi}{3}\right).$$

$$\vec{K} \parallel \vec{b}_1, \vec{b}_2, \vec{b}_3$$





$$t_{ab}(\vec{k}) = \sum_{i=1,2,3} e^{i\vec{k}\cdot\vec{a}_i} = 0 + i\delta\vec{k}\cdot\sum e^{i\vec{K}\cdot\vec{a}_i}\,\vec{a}_i$$

$$\begin{bmatrix} 0 & t_{ab}(\vec{k}) \\ t_{ba}(\vec{k}) & 0 \end{bmatrix} = \begin{bmatrix} 0, & \Pi_1 + i\Pi_2 \\ \Pi_1 - i\Pi_2, & 0 \end{bmatrix}$$

$$t_{ab}(\vec{k}) \cong \Pi_1 + i\Pi_2 \quad \text{near } \vec{K}$$

$$t'_{ab}(\vec{k}) \cong -\Pi_1 + i\Pi_2 \quad \text{near } \vec{K}'$$

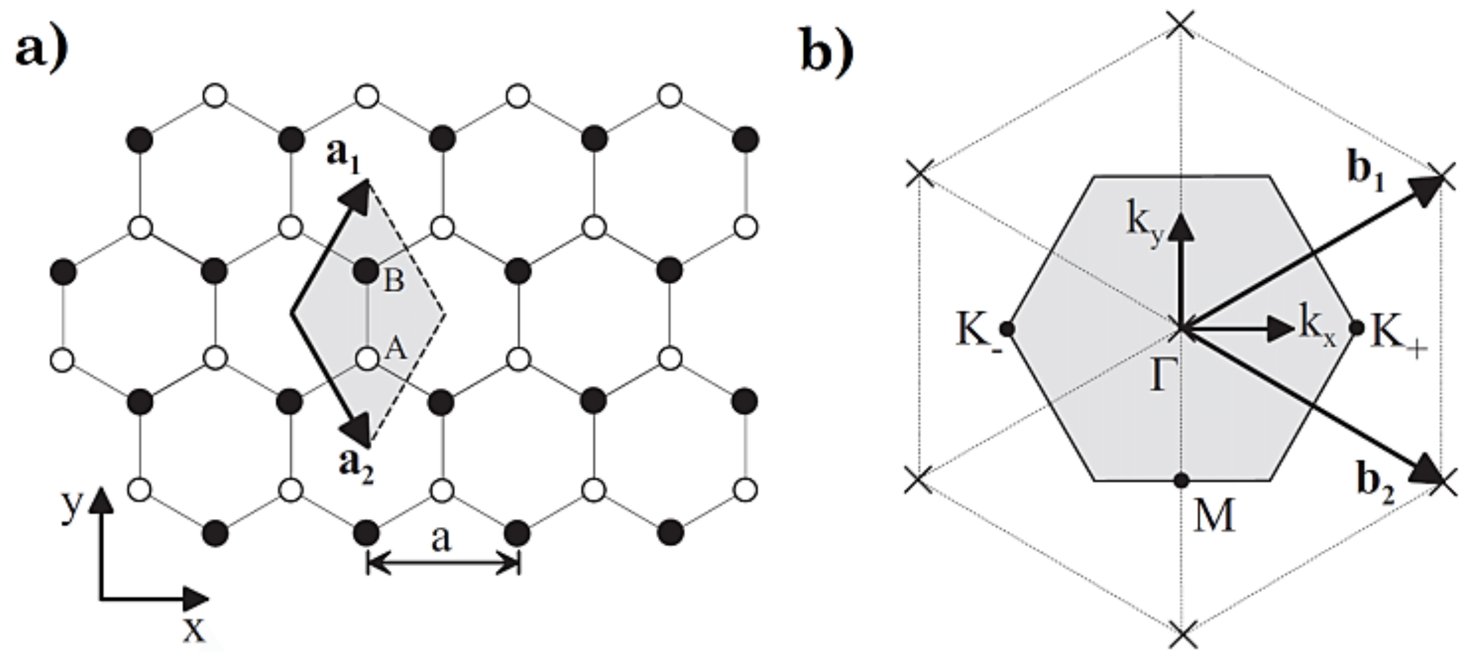
$$t_{ab}(\vec{k}) = 0 + i\,\delta\vec{k} \cdot \sum_i e^{\pm i \vec{K}\cdot\vec{a}_i}\,\vec{a}_i$$

$$t_{ab}(\delta\vec{k}) = -t^{*\prime}_{ab}(\delta\vec{k})$$

Near $\vec{K}$ Near $\vec{K}'$

$$\mathcal{H}(p, m) = \tau_z \otimes \sigma_x p_x + \sigma_y p_y + m \sigma_z$$

τ_z : Valley Subspace ; $\sigma_{x,y,z}$: Sublattice subspace



PHC Transformation:

$$C = K\,\tau_z \otimes \sigma_x$$

K → complex Conjugate

$$C^{+} \mathcal{H}(p, m)\, C = -\mathcal{H}(-p, m)$$

Time Reversal Transformation:

$$\mathcal{T} = K\,\tau_x\,, \qquad \mathcal{T}^{+} \mathcal{H}(p.m)\, \mathcal{T} = +\mathcal{H}(-p, m)$$

Chiral Transformation

$$S = i\,\tau_y \otimes \sigma_x \qquad S^{+} \mathcal{H}(p.m)\, \mathcal{T} = -\mathcal{H}(p.m)$$

more interesting case: PHC symmetry without U(1) symmetry via SSB