Phys525:
Quantum Condensed Matter Physics:

Episode three: PHC as an internal emergent symmetries (ES):
crystal effects, SSB and slightly formal stuff

PHC/CC symmetry as an internal ES

- PHC/CC symmetry: invariant dynamics when particles (e) become holes (-e).

- PHC/CC intrinsic and fundamental to QFT (relativistic) but not to CMP. but in CMP, it can emerge naturally in lattices or without lattices.

- PHC important for EFT studies of interactions as well as topological classifications.

- The only internal ES!! (well there is a close cousin).

- Emergent PHC/CC symmetries: case analysis

- A) due to interactions with background crystal structures, i.e. band structures with U(1) charge symmetries. (Often need tuning!!)

- B) due to mutual interactions between particles via spontaneous symmetry breaking. (Very robust!!)

- Continue our discussions on the simple Model (with emergent PHC symmetry along with relativistic space-time symmetry).

- Some more formal discussions on PH transformations and basic algebraic structures.

- More realistic the Haldane lattice model

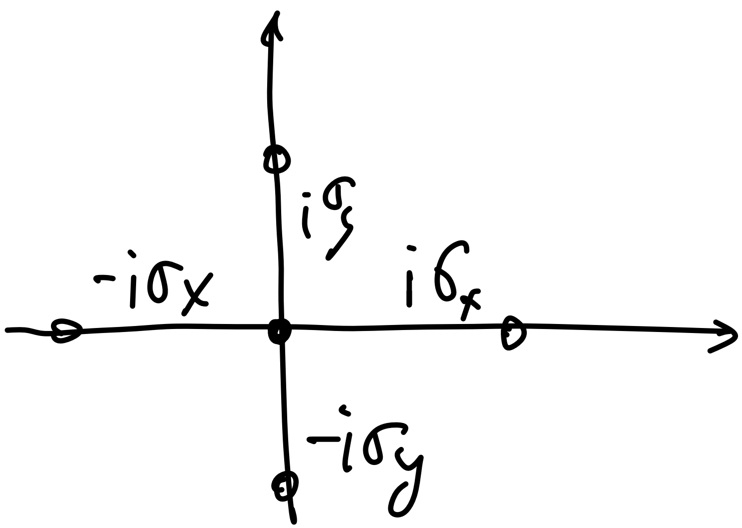
Three Examples of internal EB of PHC

- Two lattice Models with PHC symmetry
- $$H_1 = -\Delta \sum_i \psi_i^\dagger \sigma_z \psi_i - t \sum_{i,\alpha} \psi_i^\dagger \sigma_z \psi_{i+\alpha} + h.c.$$
- $$H_2 = -t \sum_{i,\alpha} \psi_i^\dagger \Gamma_\alpha \psi_{i+\alpha} + h.c., \Gamma_\alpha = -\Gamma_{-\alpha} = i\vec{\sigma} \cdot \overrightarrow{\alpha}$$

$$H_2 = -t \sum_{i,\alpha \in NN} \psi_i^\dagger \Gamma_\alpha \psi_{i+\alpha} + h.c., \quad \alpha \in NN$$

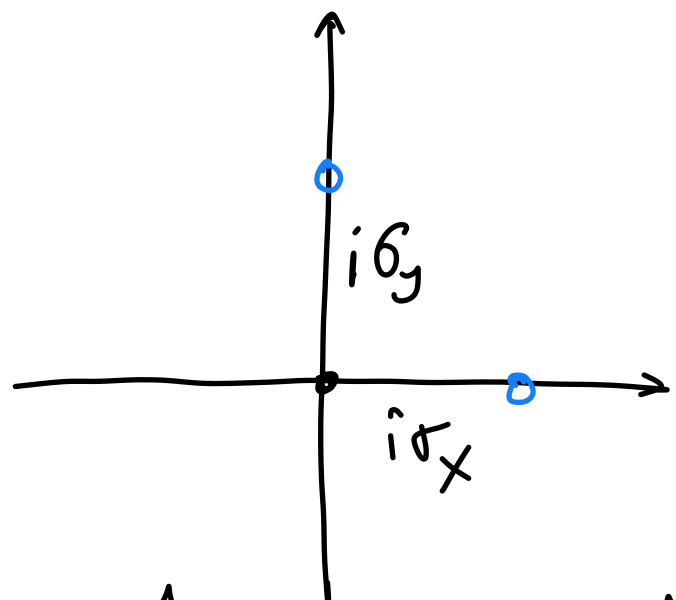
$$= -t \sum_{i,\hat{e}_\alpha} \psi_i^\dagger \Gamma_{\hat{e}_\alpha} \psi_{i+\hat{e}_\alpha} + h.c.$$

$\alpha = x, y, z$



$\alpha \in NN$

or



$\hat{e}_\alpha = \hat{e}_x, \hat{e}_y, \hat{e}_z$

PHC symmetry with U(1) symmetry

$$h_k(\sigma) = \sigma_x \sin k_x + \sigma_y \sin k_y + \sigma_z \sin k_z, H_2 = -2t \sum_k \psi_k^\dagger h_k \psi_k$$

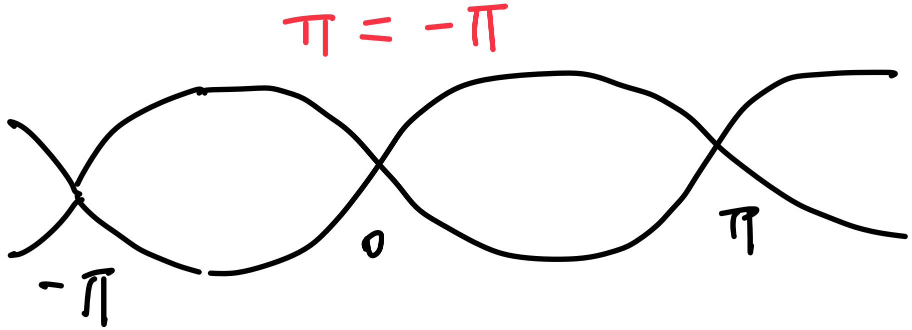
However, PHC symmetry broken if

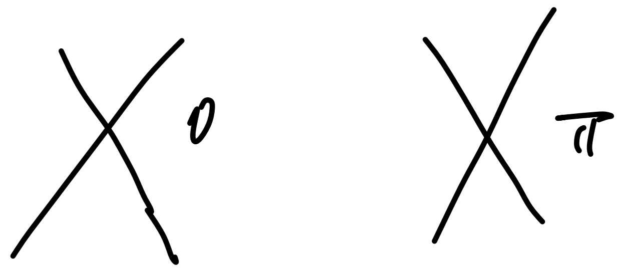
$$\Gamma_\alpha' = \Gamma_\alpha + \delta\Gamma_\alpha, \quad \delta\Gamma_\alpha = \delta \mathbb{1}$$

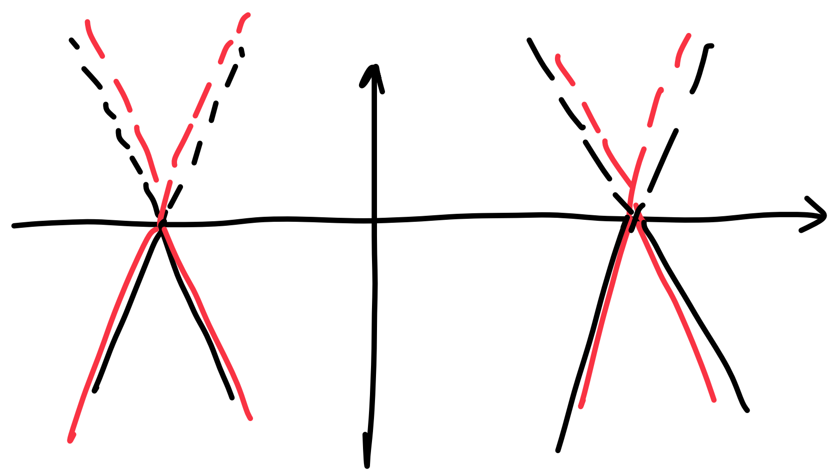
Γ_α ✓: $\Gamma_\alpha = -\Gamma_{-\alpha} = -\Gamma_\alpha^\dagger$

$\mathbb{1}$ ↗ unitary

H_2

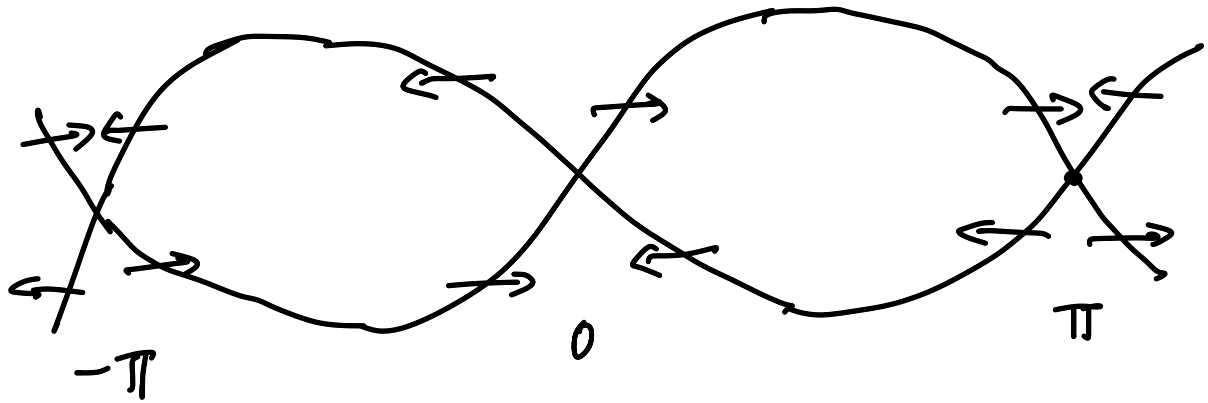






"PHC Symmetry" with U(1) Symmetry

$$k_{\pi} = k_{-\pi}$$



- Nielsen-Ninomiya theorem of fermion doubling

- H.B. Nielsen and M. Ninomiya. Absence of neutrinos on a lattice: (i). proof by homotopy theory. Nuclear Physics B, 185(1):20–40, 1981; (ii). intuitive topological proof. Nuclear Physics B, 193(1):173–194, 1981.

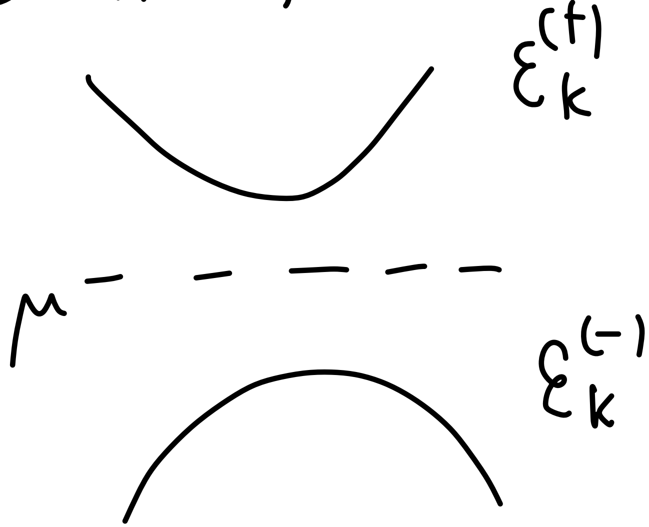
- A Weyl fermion cone (left or right) is half of Dirac fermion cone (massless).

- On lattices with U(1) symmetry, Weyl fermion cones always appear in pairs.

- On lattice with U(1) symmetry and Chiral symmetry (U(1) x U(1)), Dirac cones always appear in pairs.

- Wilson fermions has only one Dirac cone at low energies but paired at high energies.

C - Transformation



$$b^\dagger_{k,+} \rightarrow b_{-k,-}$$

$$b_{k,+} \rightarrow b^\dagger_{-k,-}$$

$H \rightarrow H$ if $\varepsilon_k^{(+)} = -\varepsilon_{-k}^{(-)}$

More formally, $C^\dagger H(p, m)\, C = -\mathcal{H}(-p, m)$

$$H = \sum_k \varepsilon_k^{(+)} b^\dagger_{k,+} b_{k,+} + \varepsilon_k^{(-)} b^\dagger_{k,-} b_{k,-}$$

$$\left[= \sum_k \varepsilon_k^{(+)} b^\dagger_{k,+} b_{k+} - \varepsilon_{-k}^{(-)} b_{-k,-} b^\dagger_{-k,-} \right]$$

$b_{-k,-} b^\dagger_{-k,-} \rightarrow a^\dagger_{k,+} a_{k,+}$

- More technical discussions on PHC transformation and PHC symmetry as supplementary materials.