Phys525:
Quantum Condensed Matter Physics:

Episode Two: Origin of emergent symmetries and why emergent symmetries (ES)

An overview of ES

- A) Emergent symmetries due to band structures (for electrons, via interactions with background periodical ions in crystals or SSB states)
- B1) Emergent symmetries due to particle-particle interactions via SSB states of particles not crystal structures.
- B2) Emergent symmetries due to strong interactions without SSB.

Type of emergent symmetries

- A) space-time symmetries (Either due to crystal structures or particle-particle interactions);
- B1) internal ES (either due to crystal structures or via interactions that lead to SSB);
- B2) “gauged symmetries”—-Emergent gauge fields, typically in topologically ordered spin liquids with anyons.

Why shall we care

- All correlations depend on emergent space-time symmetries and hence quantum dynamics.
- Universality of strong coupling physics near Fixed points crucially depend on ES both internal and space-time ESs.
- Topological states/topologies depend on internal ES.

- Emergent PHC symmetries: case analysis

- A) due to interactions with background crystal structures, i.e. band structures with U(1) charge symmetries.

- B) due to mutual interactions between particles via spontaneous symmetry breaking.

Three Examples of internal EB of PHC

- Two lattice Models with PHC symmetry
- $$H_1 = -\Delta \sum_i \psi_i^\dagger \psi_i - t \sum_{i,\alpha} \psi_i^\dagger \sigma_z \psi_{i+\alpha} + h.c.$$
- $$H_2 = -t \sum_{i,\alpha} \psi_i^\dagger \Gamma_\alpha \psi_{i+\alpha} + h.c., \Gamma_\alpha = -\Gamma_{-\alpha} = i\vec{\sigma} \cdot \overrightarrow{\alpha}$$

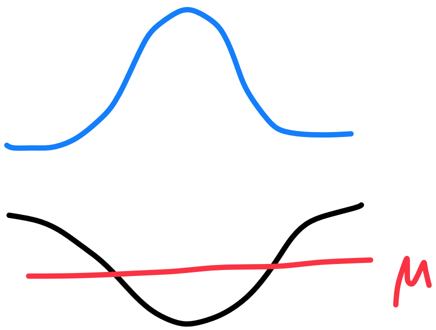
PHC symmetry with U(1) symmetry

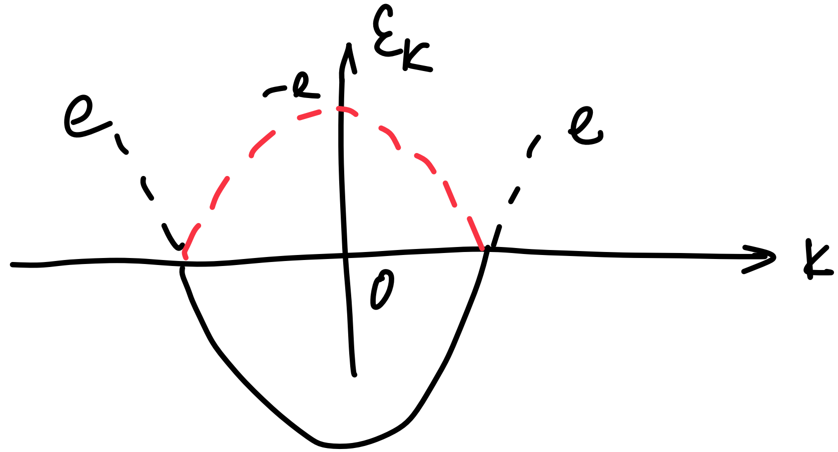
$$h_k(\sigma) = \sigma_x \sin k_x + \sigma_y \sin k_y + \sigma_z \sin k_z, H_2 = -2t \sum_k \psi_k^\dagger h_k \psi_k$$

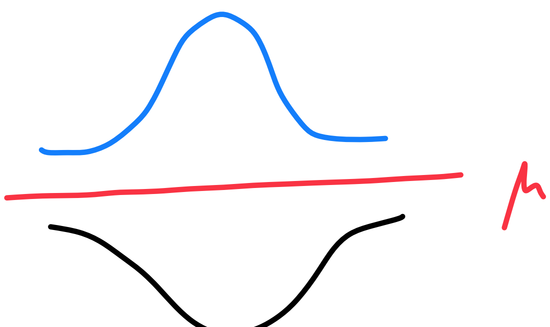
H_1 with U(1) Symmetry

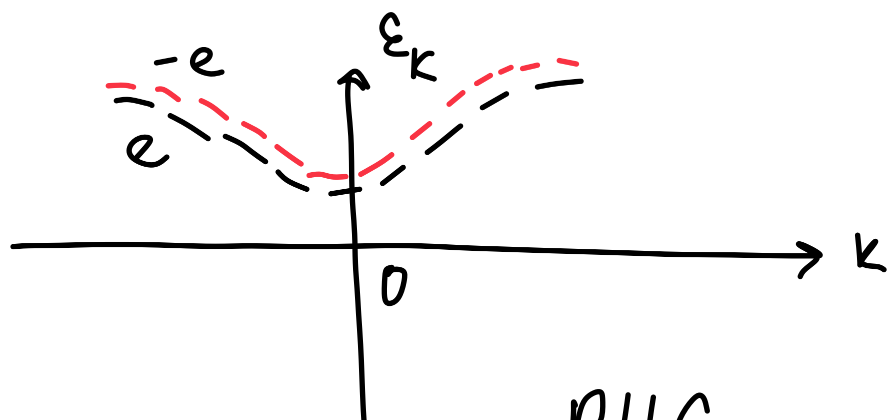
$t > 0$

$t < 0$



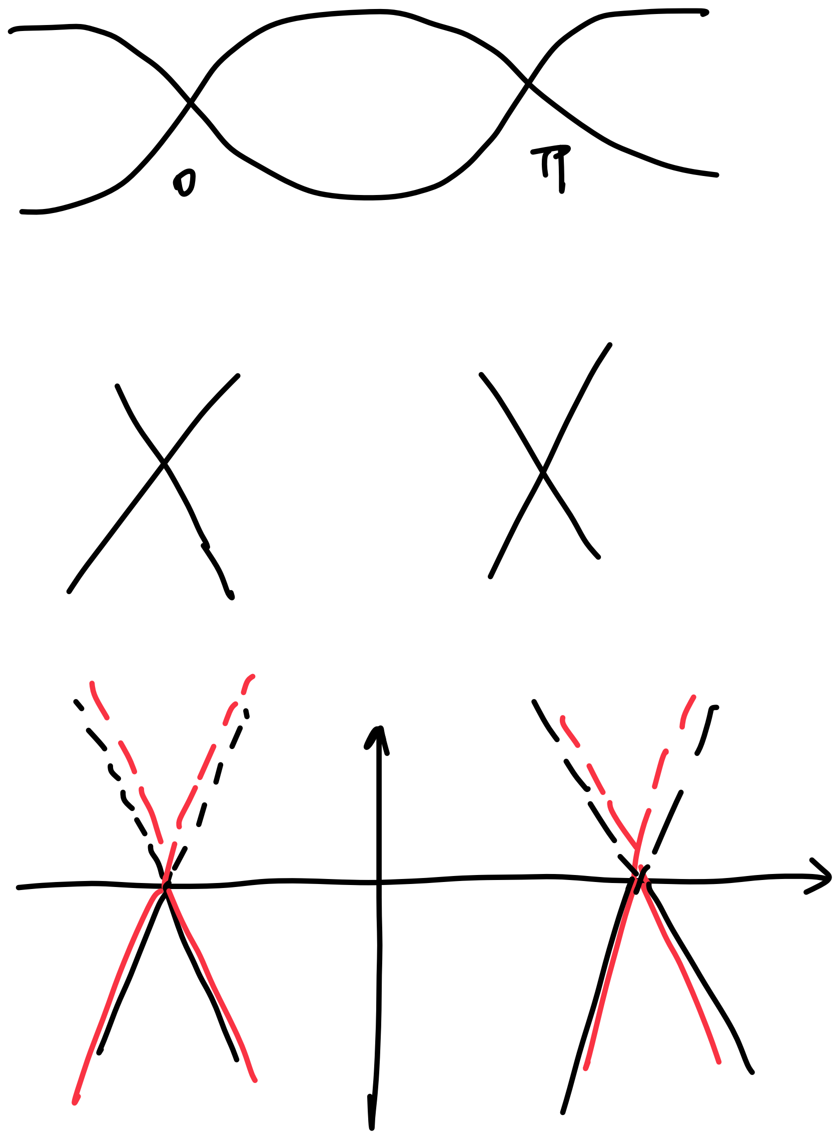






PHC

H_2



"PHC Symmetry" with U(1) Symmetry

- Nielsen-Ninomiya theorem of fermion doubling

- H.B. Nielsen and M. Ninomiya. Absence of neutrinos on a lattice: (i). proof by homotopy theory. Nuclear Physics B, 185(1):20–40, 1981; (ii). intuitive topological proof. Nuclear Physics B, 193(1):173–194, 1981.