Phys525:
Quantum Condensed Matter Physics:
emergent symmetry and phenomena

Topological States, Topological ordered states and SPT: II


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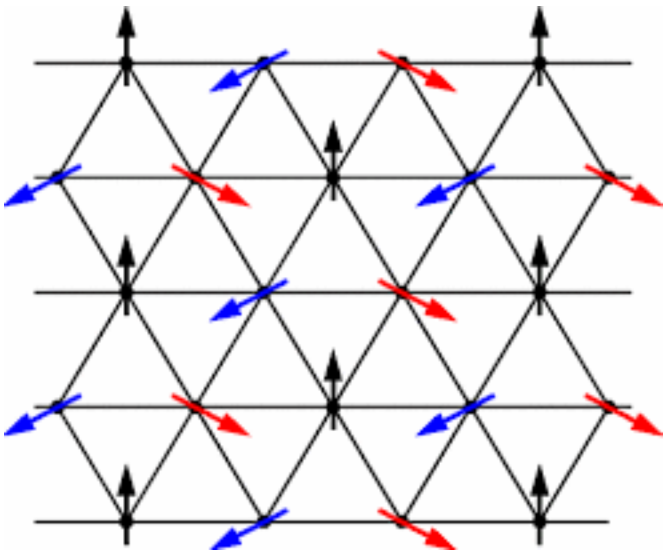
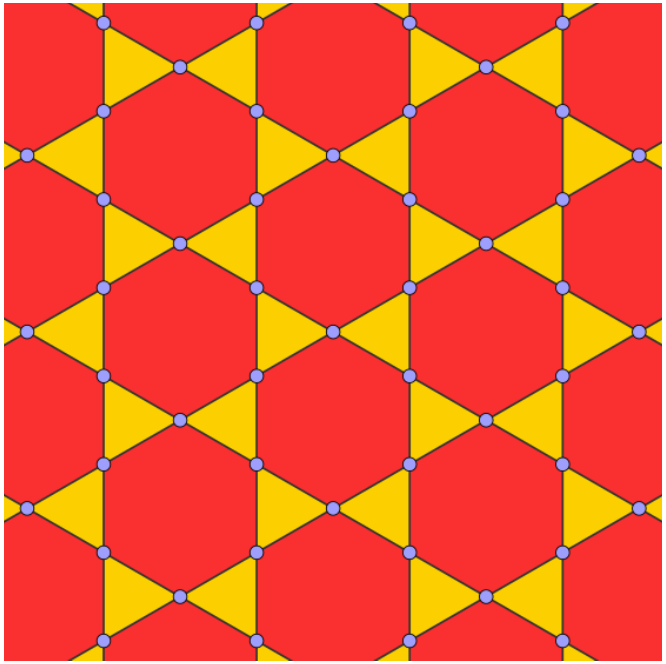
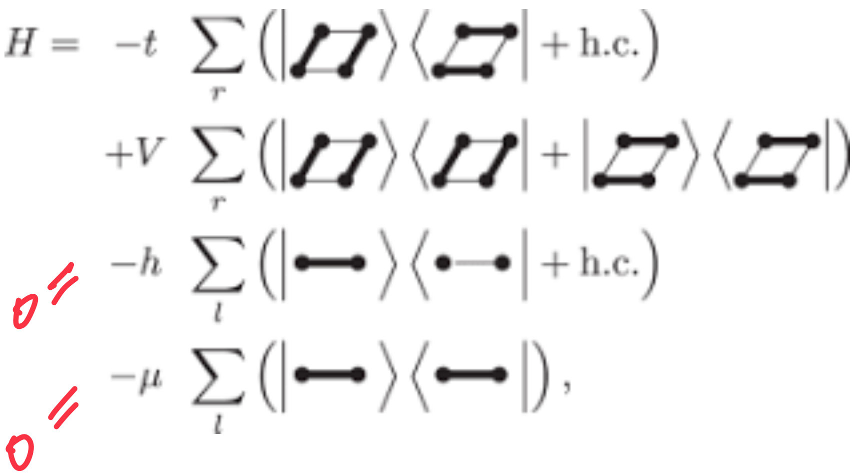
Triangle and kagaome lattices
(with SU(2) exchange interactions; Z2 spin liquids)

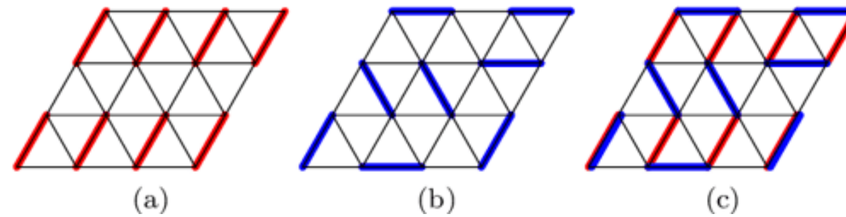


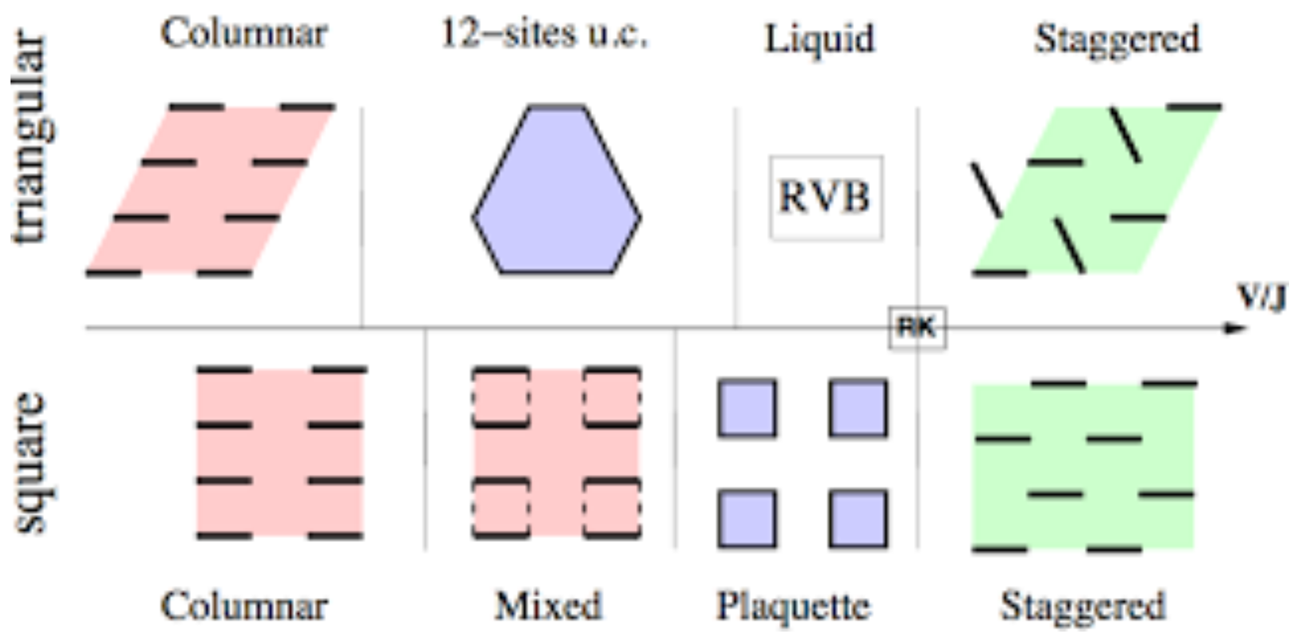


quantum dimer model (for spin liquids)

(Rokhsar, Kivelson, 89; Moessner, Sondhi, Fradkin, 2002-2003)







Affleck-Kenndy-Lieb-Tasaki, 1987 1983

$$H_{AKLT} = J \sum_{\langle ij \rangle} \left(S_i \cdot S_j + \alpha \left(S_i \cdot S_j \right)^2 \right),$$

$\alpha = 0$ Haldane

$\alpha = 1/3$, AKLT

[$S=1$, $SU(2)$] Model

bilinear-biquadratic Model

Two site solutions

$$(S_i \cdot S_j) = \frac{S^2 - S_i^2 - S_j^2}{2} = \frac{S^2}{2} - 2$$

$\alpha = 0$

- $S=2$: J
- $S=1$: $-J$
- $S=0$: $-2J$

$\alpha = \frac{1}{3}$

- $S=2$: $\frac{4J}{3}$
- $S=0$: $-\frac{2J}{3}$
- $S=1$: $-\frac{2J}{3}$

$$H_{AKLT} = 2J \sum_{\langle ij \rangle} P_{ij}^{(S=2)}, \quad J > 0$$

Representation

$|S=1\rangle$

split $\swarrow \quad \downarrow$

$|\uparrow\rangle, \; |\downarrow\rangle$

To impose $P^{(2)}_{ij}$

$$\begin{cases} |\uparrow\uparrow\rangle & S_z = 1 \\ |\downarrow\downarrow\rangle & S_z = -1 \\ \frac{1}{\sqrt{2}}\left(|\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle\right) & \end{cases}$$

$$\frac{1}{\sqrt{2}}\left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)$$ (Needs to be removed)

VBS, Valence-bond solid as the ground state

$s=0, \; s=1$ only

"Project out $|s=1\rangle$"

Elementary Properties known since 90s

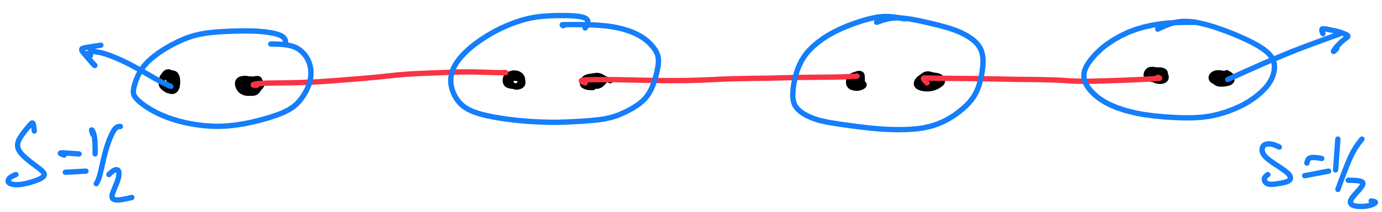
1) String order by Denijs, Rommelse (1989)

$$\langle S_i^z S_j^z \rangle \longrightarrow 0 \quad \text{as } i-j \to \infty,$$

but

$$\langle S_i^z \underbrace{e^{i\pi \sum_k S_k^z}}_{i<k<j} S_j^z \rangle \neq 0 \quad \text{as } i-j \to \infty.$$

2) Open boundary AKLT has 4-fold degeneracy.
(fractionalized)



(These features extendable to the Haldane phase.)

More Sophisticated Interpretation: Kennedy, Tasaki, 1992 on Haldane phase

$$\begin{cases} U^{+} H U = \tilde{H} \\ U^{+} S_i^z \prod_k e^{i\pi S_k^z} S_j^z U = S_i^z S_j^z \end{cases}$$

$\tilde{H}$ has $Z_2 \otimes Z_2$ symmetry, or $Z_2 \otimes Z_2 = D_2$

(π-Rotations about x, y, z axes)

Ground state 4-fold degenerate; ferromagnetic Order.

$U = \prod_{j<k} e^{i\pi S_j^z S_k^x}$, $\tilde{H}$ = TBC.

Matrix-Product-State Rep (DMRG)

(~2011, Modern studies)

$$|g.s\rangle = \sum_{\{s_i\}} \alpha_{s_1, \dots s_n} |s_1, \dots s_n\rangle, \quad s_i = 0, \pm 1$$

$$\alpha_{s_1 \dots s_n} = \mathrm{Tr}\, A^{s_1} A^{s_2} \dots A^{s_n}$$

$$A^0 = +\sqrt{1}\, \sigma^z, \quad A^+ = -\sqrt{\tfrac{2}{3}}\, \sigma^+, \quad A^- = -\sqrt{\tfrac{2}{3}}\, \sigma^-$$

(ideas coming from QI, Verstraete, Cirac, Perez-Garcia, Mung, 2008; Applications in CMT start in 2009 on; Numerical applications DMRG in 2011, Schollwock.)

Why SPT ?

(Symmetry protected topological states)

- are all spin disordered states (SU(2) spins) topologically ordered, i.e. is 1D AKLT (more discussions today) topologically ordered in the way how 2D Z_2 spin liquids are ordered (nonlocal order)?

- What happens to Z_2 TI or TSCs discussed in free fermion models with TRS when interactions are induced? (Next week).

Topological ordered states vs Symmetry protected topological states

a series of impressive attempts of using entanglement measure by Chen, Gu, Wen et.al, 2009-2012 (in Group 2)

- Topologically ordered states (TOS) with long range entanglements, or topological entropy; or topological degeneracy on torus etc. (Levin, Wen, 2005; Kitaev, Preskill, 2005)

- Symmetry protected topological states (SPT) with short range entanglement entropy; no topological degeneracy on torus etc (discussions on AKLT).

- SPT states belong to a well defined phase only if the symmetry is respected. Otherwise not. (Can be a presentation topic.)

On surfaces of SPT vs TOS

- SPT always has gapless surfaces or degenerate surfaces disregarding how you cut. To gap it, surfaces are either SSB or intrinsic topologically ordered (no contradictions to what we had before!! see next page or come back next week).

- TOS not always have gapless boundaries. ex: Z2 Spin liquids. However, it can have, say in FQHE.

What do we mean by saying “topological order” ?

Topics suggested for P3

- 1) Crossover from AKLT or other SPT into trivial phases.
- 2) Kennedy-Tasaki hidden $D_2 = Z_2 \times Z_2$symmetry in the 1D Heisenberg spin model—-extending AKLT feature to the Haldane phase.
- 3) Topological Entanglement entropy: how to choose subsystems to pick up the gamma- term/topological term —-more formal one.

Topological entropy as a measure of topological order
(Levin and Wen, 05; Kitaev and Preskill, 05)

$$S_{VN} = \alpha L - \gamma, \gamma = \ln D$$

$$D = \sqrt{\sum_a d_a^2}$$

$$\rho_A = Tr_B \rho_{AB}$$

$$S_{VN} = -Tr\rho_A ln\rho_A$$

- based on von Neumann entropy and quantum dimension D. D=2 for the toric code, $\sqrt{q}$ for FQHE with filling factor 1/q, q=3,5,7..