

Phys525:  
Quantum Condensed Matter Physics:  
emergent symmetry and phenomena

Topological States, Topological ordered states and SPT



What do we mean by saying topological matter?



- Two important concepts from 80-90s on topological order (gapped states only)
- Boundary dynamics are important; bulk-edge correspondence emerged during that time (mainly in the context of FQHE).
- “Topological degeneracy”—that ground states (with TS) can be degenerate. And if not (and if no TS symmetry breaking), non-topological.
- Affleck-Lieb-Mattis theorem on  $s=1/2$  non-topological gapped spin states: breaking

# Topological insulators and superconductors (2005~)

(free fermion classification back to 10-fold way)

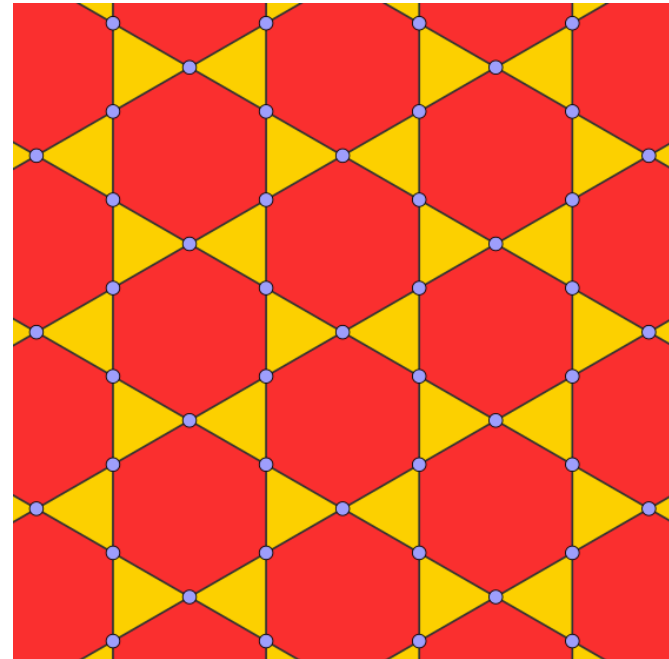
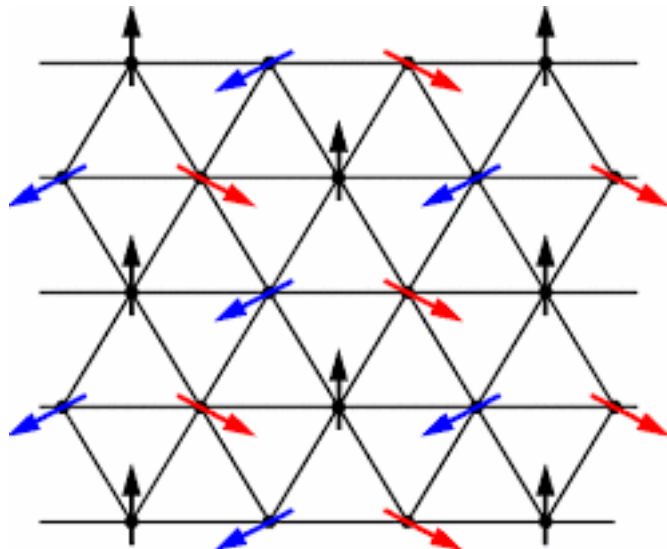
- topological insulators without symmetry or with  $Z_2$  symmetry—QHE, Haldane lattice model ( $\mathbb{Z}$ ).
- Topological Insulator with time reversal symmetry— $Z_2$  topological Insulator
- Topological superfluids without symmetry or with  $Z_2$  symmetry— $p+ip$
- topological Superconductors with time reversal symmetry— $(p+ip) \times (p-ip)$

# SPT: symmetry protected topological states (~2010 on)

- A class of states that don't have non-local order or topological order; examples include AKLT,  $Z_2$  topological insulators, topological superconductors etc. **Not including FQHE, Kitaev spin liquids that are believed to carry non-local order.**

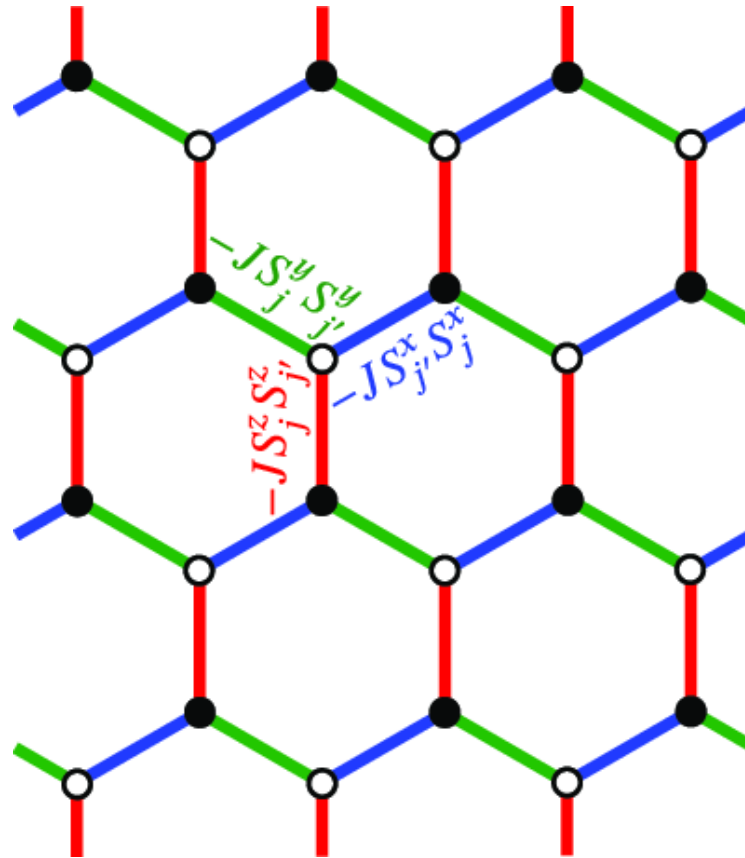
Topological Ordered states	Vs	SPT (Topological states)
FQHE		Topological Insulators
Kitaev Spin liquids		Topological Superconductors
$\mathbb{Z}_2$ - Spin liquids		AKLT states (Spin Model)

# Triangle and kagaome lattices (with $SU(2)$ exchange interactions; $Z_2$ spin liquids)



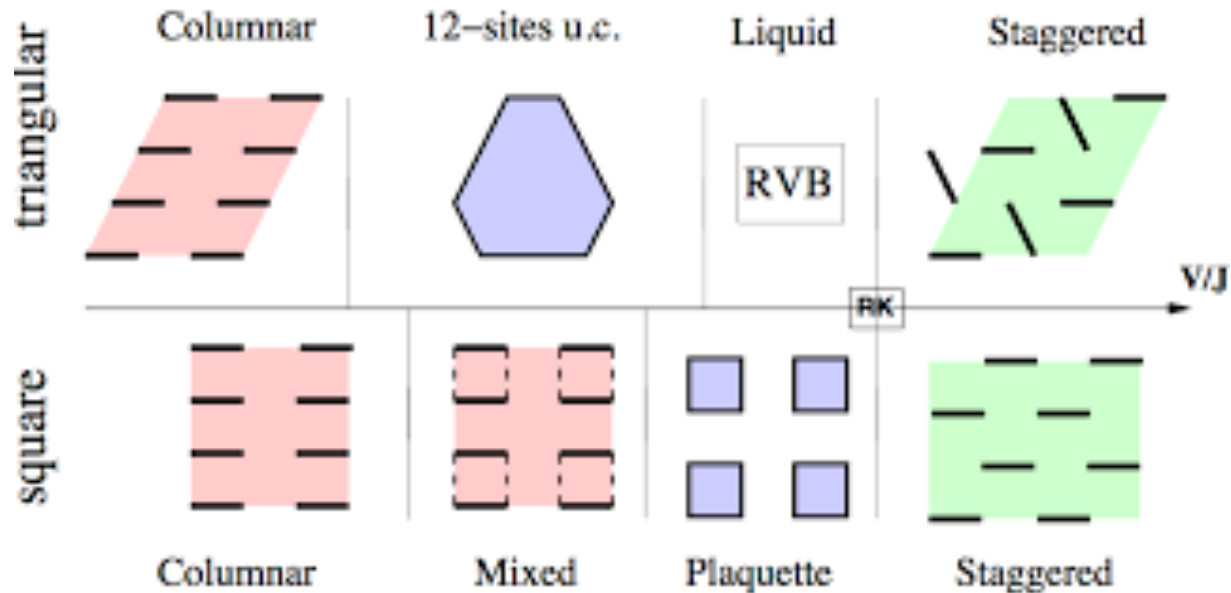
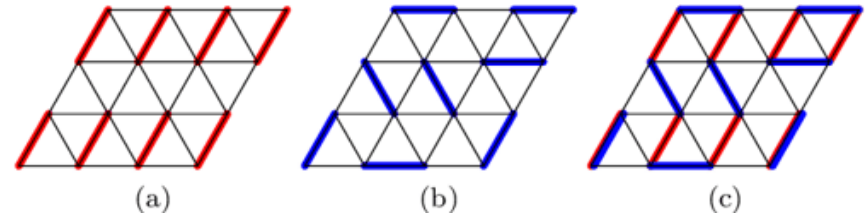


# kitaev spin liquid



# quantum dimer model

$$\begin{aligned}
 H = & -t \sum_r (|\text{dimer}\rangle \langle \text{dimer}| + \text{h.c.}) \\
 & +V \sum_r (|\text{dimer}\rangle \langle \text{dimer}| + |\text{dimer}\rangle \langle \text{dimer}|) \\
 & -h \sum_l (|\text{monomer}\rangle \langle \text{monomer}| + \text{h.c.}) \\
 & -\mu \sum_l (|\text{monomer}\rangle \langle \text{monomer}|),
 \end{aligned}$$



# Topological ordered states vs Symmetry protected topological states (~2010)

- Topologically ordered states (TOS) with long range entanglements, or topological entropy; or topological degeneracy on torus etc.
- Symmetry protected topological states (SPT) with short range entanglement entropy; no topological degeneracy on torus etc (discussions on AKLT).